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KLINGENBERG STRUCTURES AND PARTIAL DESIGNS II: REGULARITY AND UNIFORMITY

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We continue our study of the c-K-structures introduced in Part I of this paper; these are triples (ϕ, Π, Π') where $\phi: \Pi \to \Pi'$ is a well-behaved incidence structure epimorphism. This paper is concerned with uniformity and regularity of c-K-structures. We obtain connections with PBIBD's ARBD's and transversal designs.

Introduction. This paper is a continuation of our paper "Klingenberg structures and partial designs I" [6]. In the previous paper, we considered generalizations of projective Klingenberg and Hjelmslev planes (*PK*-, resp., *PH*-planes) and investigated congruence relations and solutions. The present paper is devoted to the study of the notions of uniformity and regularity.

It is well-known that a *PH*-plane Π is uniform if and only if Π induces an ordinary affine plane in each point neighborhood (Lüneburg [17]). This formulation of the idea of uniformity is herein generalized (in two ways) to c-K-structures: the induced incidence structures are now required to be "$(s, r; \mu)$-nets. These $(s, r; \mu)$-nets (which simultaneously generalize the classes of nets and affine resolvable designs) are considered in the first section of the present paper (numbered §5). These structures are, in fact, the duals of transversal designs (λ not necessarily $= 1$) which were investigated by Hanani in [10]. Uniformity and pre-uniformity are then studied in §6.

In §7, we generalize the notion of a regular *PK*-plane (Jungnickel [13], [16]) to that of a regular c-K-structure and are led thereby to the use of difference methods. We obtain a theorem characterizing the invariants of all regular, balanced, minimally uniform H-structures that is similar to the main result of Part I (Theorem 3.9). Finally, the notions of the preceding sections are combined in §8 where we study regular pre-uniform K-structures. We obtain a complete characterization of the invariants in a special case, but there remains an interesting open problem.

The notation used in this paper agrees with that of Part I and hence in general with that of Dembowski [5]. We decided to continue the numbering of Part I: thus the first section of the paper is called §5, and references like "Theorem 3.9" or "(2.5)" refer without saying to Part I.