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SYMMETRIC SUBLATTICES OF A NOETHER LATTICE

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In this note we investigate questions about partitions of positive integers arising from multiplicative lattice theory and prove that the sublattice of $RL(A_i)$ ($A_1, \cdots, A_k$ is a prime sequence in a local Noether lattice) generated by the elementary symmetric elements in the A_i's is a π-lattice.

0. Introduction. If $A_1, A_2, \cdots, A_k$ is a prime sequence in L, a local Noether lattice, then the multiplicative sublattice it generates is isomorphic to RL_k, the distributive local Noether lattice with altitude k. We denote this sublattice of L by $RL(A_i)$. In $RL(A_i)$, every element is a finite join of products $A_1^{r_1}A_2^{r_2}\cdots A_k^{r_k}$ for $(r_1, \cdots, r_k) = (r_i)$ a k-tuple of nonnegative integers. Minimal bases for an element, T, in $RL(A_i)$ are unique and determined by the exponent k-tuples of the elements in the minimal base for T. We examine the sublattice of L generated by the elementary symmetric elements in the prime sequence $A_1, \cdots, A_k$. This multiplicative sublattice is a π-domain (Theorem 7.1).

Unless otherwise stated, all k-tuples will be nonnegative integers. A k-tuple (r_i) is *monotone* if and only if $r_i \geqq r_{i+1}$ for $1 \geqq i > k$. $(r_i) = (s_i)$ and $(r_i) + (s_i)$ refer to componentwise equality and addition respectively. $(r_i) \geqq_p (s_i)$ means $r_i \geqq s_i$ for $i = 1, \cdots, k$. We write $(r_i) \geqq_l (s_i)$ to mean the first nonzero entry in $(r_i - s_i)$ is strictly positive (lexicographic order). If (e_i) is a k-tuple we write e_i^* for $\sum_{j=i}^k e_j$ and e_i^{**} for $\sum_{j=i}^k e_j^*$. Throughout this note $A_1, \cdots, A_k$ is a prime sequence in L and $RL(A_i)$ is the multiplicative sublattice it generates.

1. The symmetric sublattice. If T is a principal element in $RL(A_i)$ and g is in S_k, the permutation group on $1, \cdots, k$, we define $T_g(T^g)$ to be the principal element in $RL(A_i)$ obtained by replacing $A_i^{t(i)}$ by the factor $A_{g(i)}^{t(i)}(A_i^{t(g(i))})$ in T for each i from 1 to k. If $C_1 \vee \cdots \vee C_p$ is a minimal base for C in $RL(A_i)$, then $C_g = (C_1)_g \vee \cdots \vee (C_p)_g$. C^g is defined similarly. Note that for each g in S_k and for C in $RL(A_i)$, $(C_g)^g = (C^g)_g = C$. Hence $C_g = C^{g^{-1}}$. An element C in $RL(A_i)$ is a *symmetric element* if and only if $C_g = C$ for each g in S_k.

THEOREM 1.1. *The set of all symmetric elements in $RL(A_i)$ forms a multiplicative sublattice of $RL(A_i)$ which is closed under residuation.*