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SMOOTH G-MANIFOLDS AS COLLECTIONS OF FIBER BUNDLES

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This paper is about the general theory of differentiable actions of compact Lie groups. Let G be a compact Lie group acting smoothly on a manifold M. Both M and M/G have natural stratifications, and M/G inherits a "smooth structure" from M. The map $M \to M/G$ exhibits many of the properties of a smooth fiber bundle. For example, it is proved that a smooth G-manifold can be pulled back via a "weakly stratified" map of orbit spaces. Also, it is well-known (and obvious) that a smooth G-manifold is determined by a certain collection of fiber bundles together with some attaching data. Several precise formulations of this observation are given.

Introduction. We develop some elementary ideas in what might be termed "the bundle theoretical aspect" of compact transformation groups. Suppose that a compact Lie group G acts smoothly on a manifold M. First consider the case where this action has only one type of orbit, that is, where all the isotropy groups are conjugate. In this case, it follows from the Differentiable Slice Theorem that M/G is a smooth manifold and that M is a smooth fiber bundle over M/G. When the action has more than one type of orbit; this is no longer true; however, there are two related points of view.

The first of these is to regard the smooth G-manifold M together with the natural projection $\pi: M \to M/G$ as a prototypical example of a "singular fiber bundle." As such, one might expect smooth G-manifolds to have many of the formal properties of ordinary fiber bundles. In an appropriate context, this is true (as we shall see in Chapter III). One of the main purposes of this paper is to describe this context.

The second point of view is to regard M as a certain collection of fiber bundles. Here the basic idea is to consider all those points in M of a given orbit type (that is, all those points with isotropy groups conjugate to a given subgroup of G). It follows from the Differentiable Slice Theorem, again, that the union of such points is an invariant submanifold of M and, therefore, a smooth fiber bundle over its image in M/G. Thus, M is a union of various fiber bundles.

Our actual approach is a slight modification of this. In Chapter I, we define a notion of "normal orbit type," which is better suited to the study of smooth actions than is the notion of orbit type. The normal orbit type of a point takes into account the slice repre-