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THE NUMBER OF NONFREE COMPONENTS IN THE DECOMPOSITION OF SYMMETRIC POWERS IN CHARACTERISTIC p

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If G is the group with p (=prime) elements and k a field of characteristic p let $V_1, V_2, \cdots, V_p$ denote the indecomposable $k[G]$-modules of k-dimension $1, 2, \cdots, p$ respectively. Let $e_{n,\nu}$ denote the number of nonfree components of the decomposition of the symmetric power $S^\nu V_{n+1}$. Then the following symmetry relation is proved

$$e_{n,p-n-\nu-1} = e_{n,\nu} .$$

As a corollary we find that $S^r V_{n+1}$ has exactly one nonfree component when $n + r = p - 2$ thus solving a problem in a previous paper by R. Fossum and the author. An explicit formula for $e_{n,\nu}$ expressed in numbers of restricted partitions is obtained.

Let G be the group with p elements where p is a prime number. Let k be a field of characteristic p. Then there are p indecomposable $k[G]$-modules $V_1, V_2, \cdots, V_p$ where

$$V_n \cong k[x]/(x-1)^n .$$

Note that $V_p = k[G]$ is free and $\dim_k V_n = n$.

The symmetric power $S^\nu V_{n+1}$ taken over k is again a $k[G]$-module and can be decomposed into a direct sum of the V_i: s

$$S^\nu V_{n+1} = \bigoplus_{j=1}^{p} c_{\nu,j}(n) V_j$$

where the integer $c_{\nu,j}(n)$ is the number of times V_j is repeated. Let

$$e_{n,\nu} = \sum_{j=1}^{p-1} c_{\nu,j}(n)$$

be the number of nonfree components in $S^\nu V_{n+1}$.

If we write down these numbers in triangular form we get the following pictures where the number in the $(\nu + 1)$th place in the $(n + 1)$th row from below is $e_{n,\nu}$.