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UNIFORM SUBGROUPS AND ERGODIC ACTIONS OF EXPONENTIAL LIE GROUPS

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We show that the restriction of an ergodic action of an exponential solvable Lie group to a uniform subgroup is still ergodic provided that the restriction to the commutator subgroup is ergodic. This complements similar results previously obtained for semi-simple and nilpotent Lie groups.

1. Introduction. Suppose G is a locally compact group which acts ergodically on a standard Borel space with quasi-invariant measure (S, μ), and that H is a closed subgroup of G. An important problem in ergodic theory is to determine when the restriction of the G-action to H is still ergodic. For certain G and H answers are known completely. For example, let G be a noncompact connected simple Lie group with finite center and $H = \Gamma$ a lattice subgroup in G. If S is a transitive G-space, then C. C. Moore [6] has shown that Γ will be ergodic on S if and only if the stability groups of the G-action are not compact. If S is properly ergodic; i.e., ergodic but not transitive (modulo null sets), then the author has shown in [9] that in all such cases Γ is ergodic on S. If G is a connected, simply connected, nilpotent Lie group, we also showed in [9], based on results in [8], that the restriction of an ergodic G-action to an arbitrary lattice subgroup Γ is ergodic provided that the restriction to $[G, G]$ is ergodic. The point of this paper is to provide a similar result for exponential solvable Lie groups. By a uniform subgroup of G we mean one whose quotient is compact with a finite G-invariant measure. Our main result is the following.

THEOREM 1. *Suppose G is a connected, simply connected, exponential solvable Lie group. Let S be an ergodic G-space. If $[G, G]$ is also ergodic on S, then Γ is ergodic on S for every uniform subgroup $\Gamma \subset G$.*

In proving the results in [9] stated above concerning simple and nilpotent Lie groups, we made important use of certain properties of the space of orbits of unitary representations of such groups. Results of this type will be equally important for us in the present situation and these will follow from recent work of R. Howe and C. C. Moore [5].

2. Projective kernels. In this section we present some results