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THE CARATHÉODORY METRIC AND HOLOMORPHIC MAPS ON A CLASS OF WEAKLY PSEUDOCONVEX DOMAINS

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The boundary behavior of proper holomorphic maps between two smoothly bounded pseudoconvex domains in C^n is studied by means of the Carathéodory metric. The Hölder continuity of such maps is proved in case the image domain satisfies some technical conditions; these are satisfied, for example, by strictly pseudoconvex domains and convex domains with real analytic boundary.

In recent years it has become clear that pseudoconvex domains with smooth boundary may exhibit rather pathological behavior in the absence of strict pseudoconvexity (cf. the examples of Kohn and L. Nirenberg [13] and Diederich and Fornaess [4]). Therefore it might be of interest to consider conditions weaker than strict pseudoconvexity and to extend classical results to more general settings.

Investigations related to Hölder estimates for solutions of the $\bar{\partial}$-equation (cf. Range [18]) have led the author to introduce a technical refinement of the following classical condition (cf. Behnke and Thullen [1], p. 29): The domain D is called *totally pseudoconvex* at $P \in \partial D$ if there is an analytic hypersurface M_P in a neighborhood U of P, such that $M_C \cap \bar{D} = \{P\}$. The refinement involves two parts. First, there should be supporting analytic hypersurfaces M_ζ for all points $\zeta \in \partial D$ near P, and M_ζ should depend smoothly on ζ. Next, in order to obtain estimates of some sort, one needs finite order contact between M_ζ and ∂D at ζ. The resulting condition is called *uniform total pseudoconvexity of finite order* (cf. Definition 1.8 for the precise formulation). Simple examples of domains which satisfy this condition at every boundary point are strictly pseudoconvex domains and convex domains with real analytic boundary.

In this paper we prove the following generalization of a classical result.

MAIN THEOREM. *Let D_1 and D_2 be bounded domains in $\boldsymbol{C}^n$ with smooth boundary. Assume that D_2 is uniformly totally pseudoconvex of finite order at every point $P \in \partial D_2$, and that $\bar{D}_2$ has a Stein neighborhood basis.*[1] *Then there is $\alpha > 0$, such that every proper*

[1] Theorem 2.2 and, as a consequence, the Main Theorem, are valid without assuming the existence of a Stein neighborhood basis, provided one assumes high differentiability