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EMBEDDINGS AND BRANCHED COVERING SPACES FOR THREE AND FOUR DIMENSIONAL MANIFOLDS

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1. Introduction. The main purpose in writing this paper is to point out a connection between embeddings of manifolds and branched covering spaces of manifolds. The following theorem is a corollary to Theorems 3, 4, and 5, and can be regarded as the main result of this paper.

THEOREM. *Let* $p: M^n \to S^n$, $n = 3$ *or* 4, *be a 3-fold dihedral branched covering space branched over a polyhedral knot or link if* $n = 3$, *or a closed orientable polyhedral surface, if* $n = 4$.

Then there is a locally flat embedding $e: M^n \to S^n \times S^2$ such that the following diagram commutes.

$$\begin{array}{ccc} & & S^n \times S^2 \\ & \overset{e}{\nearrow} & \downarrow \\ M^n & \xrightarrow{\ p\ } & S^n \end{array}$$

It is a result of the author and José M. Montesinos ([2], [5]) that every closed orientable 3-manifold is a three fold dihedral covering of S^3 branched over a knot or link. Indeed, this can be done in a wide variety of ways satisfying various side conditions ([3]).

This result, together with the above theorem can be viewed as saying that every closed orientable 3-manifold and certain closed orientable 4-manifolds are topologically like Riemann surfaces.

Indeed, given such an $M^{3 \text{ or } 4}$ there is an S^2 multivalued function f (see § 4) defined on $S^{3 \text{ or } 4}$ such that $M^{3 \text{ or } 4}$ is the graph of f. Moreover, locally the singularities of f look like $(x, z) \to \sqrt{z}$ or $(x_1, x_2, z) \to \sqrt{z}$.

It is unknown which closed orientable 4-manifolds can be 3-fold dihedral covering spaces of S^4 branched over orientable surfaces. But Montesinos ([7]) has recently shown that a large and important class of four manifolds with boundary are three fold dihedral coverings of D^4, branched over locally flat, but not necessarily orientable, properly embedded surfaces. On the other hand, it is a result of Edmonds and Berstein that $S^1 \times S^1 \times S^1 \times S^1$ and many other closed orientable four manifolds cannot be threefold branched covering