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ON THE DIVISORS OF MONIC POLYNOMIALS OVER A COMMUTATIVE RING

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For a commutative ring R with identity, denote by $R\langle X\rangle$ the quotient ring of $R[X]$ with respect to the regular multiplicative system S of monic polynomials over R. The present paper determines the group of units of the ring $R\langle X\rangle$. This is equivalent to the problem of determining the saturation S^* of the multiplicative system S; by definition, S^* consists of all divisors of monic polynomials over R. For a nonzero polynomial

$$f = f_0 + f_1X + \cdots + f_nX^n \in R[X] ,$$

it is shown that each of the following conditions (A) and (B) is equivalent to the condition that f divides a monic polynomial over R (in (B), the ring R is reduced).

(A) The coefficients of f generate the unit ideal of R and, for each j between 0 and n and for each prime ideal P of R, the relations $f_{j+1}, \cdots, f_n \in P$, $f_j \notin P$, imply that f_j is a unit modulo P.

(B) There exists a direct sum decomposition

$$R = R_1 \oplus \cdots \oplus R_m \quad \text{of} \quad R$$

such that if $f = g_1 + \cdots + g_m$ is the decomposition of f with respect to the induced decomposition

$$R[X] = R_1[X] \oplus \cdots \oplus R_m[X] \quad \text{of} \quad R[X] ,$$

then the leading coefficient of g_i is a unit of R_i for each i.

One corollary to the preceding characterizations is that S^* is the set of polynomials over R with unit leading coefficient if and only if the ring R is reduced and indecomposable.

Let R be a commutative ring with identity, let X be an indeterminate over R, and denote by S the regular multiplicative system of monic polynomials over R. The quotient ring $R[X]_S$ of $R[X]$ is currently receiving attention, probably because of the role it plays in Quillen's solution of the Serre Conjecture [6]. We use the symbol $R\langle X\rangle$ to denote the ring $R[X]_S$; this differs from Quillen's notation $R(X)$ for this ring, but our choice of notation is based on the fact that $R(X)$ has traditionally been used to denote the quotient ring of $R[X]$ with respect to the multiplicative system of polynomials of unit content [5, p. 18], [2, §33]. The aim of this paper is to determine the group of units of the ring $R\langle X\rangle$. If $S^* = \{f \in R[X] \mid f$ divides an element of $S\}$ (that is, S^* is the *saturation*