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MULTIPLICATIVE LINEAR FUNCTIONALS OF STEIN ALGEBRAS

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Let $(X, \mathcal{O}_X)$ be a Stein analytic space, and let $\mathcal{O}(X)$ denote the space of global sections of $\mathcal{O}_X$ endowed with its usual Frechet topology. The question of the continuity of complex valued multiplicative linear functionals of $\mathcal{O}(X)$ will be studied. The main result can be stated as follows: Theorem: Let $(X, \mathcal{O}_X)$ be a Stein space, and let $\alpha: \mathcal{O}(X) \rightarrow C$ be a multiplicative linear functional. Suppose one can find an analytic subset $Y \subset X$ such that all the connected components of both Y and $X-Y$ are finite dimensional. Then α must be continuous. More generally, suppose that one can find a sequence of analytic subsets of X, $X=Y_0 \supset Y_1 \supset \cdots \supset Y_n=\emptyset$, such that for any i, $0 \leqq i < n$, all the connected components of $Y_i - Y_{i+1}$ are finite dimensional. Then α must be continuous.

This paper resulted from an attempt to understand the claim made without proof in [5] that if $(X, \mathcal{O}_X)$ is a Stein space, and if $\lambda: (X, \mathcal{O}_X) \rightarrow \text{Spec}\ (\mathcal{O}(X))$ is the natural morphism, then the pair $((X, \mathcal{O}_X), \lambda)$ is an analytic C-cover of Spec$(\mathcal{O}(X))$. (See [5] for definitions.) In particular, all multiplicative linear functionals of $\mathcal{O}(X)$ would have to be continuous for this to be true. Michael proved the continuity of such functionals in case X is a domain of holomorphy in C^n [7]. (He in fact conjectured the continuity of all multiplicative linear functionals on any Frechet algebra [7].) A result of Arens [1] guarantees the desired continuity in case X can be embedded as a closed subspace of some C^n. Forster [3] proved the desired continuity in case X is finite dimensional. My result is a generalization of Forster's. Markoe [6] gave a weaker extension of Forster's result. He showed continuity under the assumption that $Sg(X)$, the singular locus of X, is finite dimensional. This follows from my result with $Y_1 = Sg(X)$ and $n = 2$. Finally, let me note that an advantage of the techniques of this paper is that they expose the elementary nature of Forster's theorem. They provide a proof which, unlike those in [3] and [6], does not depend on the deep existence of a proper map from a finite dimensional Stein space to some Euclidean space.

1. Preliminaries. Let X be a Stein space. (In what follows I will write X rather than $(X, \mathcal{O}_X)$ for analytic spaces as long as this leads to no ambiguity.) If $\mathcal{F}$ is a coherent analytic sheaf on