PACIFIC JOURNAL OF MATHEMATICS
Vol. 78, No. 1, 1978

THE FACTORS OF THE RAMIFICATION SEQUENCE OF A CLASS OF WILDLY RAMIFIED v-RINGS

ROBERT D. DAVIS

Let R_e denote a v-ring of characteristic 0, with maximal ideal M and residue field h of characteristic p $(p \neq 0, 2)$ in which p generates the eth power of the maximal ideal. If p divides e, R_e is said to be wildly ramified. This work is concerned primarily with the determination of the factor groups of the ramification sequences of wildly ramified v-rings having ramification $2p$.

The canonical homomorphisms of the ramification sequence are used to show that in all except G_1/H_1 the successive factor groups are isomorphic to subgroups of the additive group of the residue field or to subgroups of the additive group of derivations on the residue field. Then the Eisenstein polynomial of R_{2p} over R is used to determine bounds on the range of the the canonical homomorphism. One then constructs inertial automorphisms, using convergent higher derivations to establish that those bounds do, in fact, describe the range. Further, it is found that if G_1/H_1 is nontrivial, it is isomorphic to the group of order 2, and that G_1/H_1 contains the first known examples of v-rings having inertial automorphisms which are neither derivation automorphisms nor automorphisms of finite order. In addition the Galois theory of totally ramified extensions R_{pq} $(q < p)$ is treated. Necessary and sufficient conditions for R_{pq}/R to be Galois are found as well as the location of the Galois maps in the ramification sequence.

The determination of the factors of the ramification sequence extends the work of MacLane [8], Heerema [4] and Neggers [9]. The Galois theory of totally ramified extensions R_{pq} $(q < p)$ of an unramified v-ring, treated in §III generalizes the work of Wishart [13] and Davis and Wishart [1]. The convergent higher derivation used here as in the work of Heerema is completely described in [5], so a discussion of it will not be included.

In addition to evaluating the factor groups of the ramification sequence, a second object of this work was to determine the relationship of the subgroup of derivation automorphisms G_D to the ramification sequence, where $\alpha \in G_D$ if there exists a convergent higher derivation $D = \{D_j\}$ such that $\alpha = \sum_{j=0}^{\infty} D_j$. In earlier work Neggers [9, Theorems 4 and 5] has shown that for arbitrary e, if $i \geqq (e + p)/(p - 1)$, $G_i \subset G_D$ and that for $i, j \geqq (e + p)/(p - 1)$, $G_i/G_{i+1} \cong G_j/G_{j+1}$ and $H_i/G_{i+1} \cong H_j/G_{j+1}$. He also characterized these factor groups in terms of derivations [9, Theorem 6]. Until now in every