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DENTING POINTS IN B^p

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It is shown that in the weighted Bergmann space B^p of analytic functions all points of the unit sphere are denting points of the unit ball.

1. Introduction and definitions. Let Δ be the open unit disk in the complex plane ($\boldsymbol{C}$). For each fixed p in $(0, 1)$ we define a finite positive measure $d\mu(z) \equiv (1 - |z|^2)^{1/p-2}dm(z)$, where $z \in \Delta$ and $dm(z)$ is the usual Lebesgue measure on Δ. We consider the closed subspace $B^p = B^p(d\mu)$ of $L^1(d\mu)$ consisting of all functions in $L^1(d\mu)$ that are analytic on Δ. B^p is the containing Banach space of the Hardy space $H^p(\Delta)$ and indeed B^p is the Mackey completion of H^p. (See Duren, Romberg, and Shields [4] and Shapiro [7].) Let B be the closed unit ball and S the unit sphere in B^p. Although the closed unit ball of $L^1(d\mu)$ has no extreme points we shall show that the ball B has certain smoothness properties. It is not a fortiori clear that B has extreme points. However, several functional analytical properties of the space B^p are known. In particular a result of Shields and Williams [8; p. 295] shows that B^p is complemented in $L^1(d\mu)$. An argument of Lindenstrauss and Pelczynski [5; p. 248] can then be used to prove that B^p is topologically isomorphic to the sequence space l^1. It is known that l^1 (being a separable, dual space) has the Radon-Nikodym property. A good reference on the Radon-Nikodym property is Diestal and Uhl [3]. Hence, if T is a topological isomorphism of B^p onto l^1 then $TB = C$ is a bounded, closed convex subset of l^1 and as such has extreme points. In fact B is the closed, convex hull of its extreme points.

If X is a Banach space and $x \in X$ with $\|x\| = 1$ we say that x is a denting point of the unit ball of X if for each $\varepsilon > 0$ the closed convex hull of the set

$$\{y \in X: \|y\| \leqq 1 \text{ and } \|y - x\| \geqq \varepsilon\}$$

does not contain x. The Radon-Nikodym property for l^1 also guarantees that $C(=TB)$ has denting points [3; p. 25, 30] and hence there are points of $S \subseteqq B^p$ which are denting points. Finally we define strong extreme point.

DEFINITION. A point x in a Banach space X, with $\|x\| = 1$ is a strong extreme point of the unit ball of X if for each $\varepsilon > 0$ there is a $\delta > 0$ such that