PACIFIC JOURNAL OF MATHEMATICS
Vol. 78, No. 1, 1978

# THE 2-CLASS GROUP OF BIQUADRATIC FIELDS, II

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**We describe methods for determining the exact power of 2 dividing the class number of certain cyclic biquadratic number fields. In a recent article, we developed a relative genus theory for cyclic biquadratic fields whose quadratic subfields have odd class number; we considered the case in which the quadratic subfield is $Q(\sqrt{l})$ with $l \equiv 5 \pmod 8$ a prime. Here we shall extend our methods to the cases in which the subfield is $Q(\sqrt{2})$ or $Q(\sqrt{l})$ with $l \equiv 1 \pmod 8$ a prime. We consider all such cases for which the 2-class group of the biquadratic field is of rank at most 3.**

## 2. Notation and preliminaries.

$Q$: the field of rational numbers.
$l$: a rational prime satisfying $l = 2$ or $l \equiv 1 \pmod 8$.
$p, q, p_i$: rational primes.
$k$: the quadratic field $Q(\sqrt{l})$.
$\varepsilon = (u + v\sqrt{l})/2$, the fundamental unit of $k$, with $u, v > 0$.
$m$: a square-free positive rational integer, relatively prime to $l$.
$d = -m\sqrt{l}\,\varepsilon$.
$K$: the biquadratic field $k(\sqrt{d})$.
$h, h_0$: the class numbers of $K$ and $k$, respectively.
$\left(\frac{x, y}{\pi}\right)$: the quadratic norm residue symbol over $k$.
$\left[\frac{\alpha}{\beta}\right]$: the quadratic residue symbol for $k$.
$\left(\frac{a}{b}\right)$: the rational quadratic residue (Legendre) symbol.
$\left(\frac{a}{b}\right)_4$: the rational 4th power residue symbol (defined if and only if $(a/b) = 1$).
$N(\ )$: the relative norm for $K/k$.
$H$: the 2-Sylow subgroup of the class group of $K$.

It is easy to see that $K$ is a cyclic extension of $Q$ of degree 4 which contains $k$. Recall that $\varepsilon$ has (absolute) norm $-1$, that $h_0$ is odd and that $H$ has rank $t-1$, where $t$ is the number of prime ideals of $k$ which ramify in $K$.

## 3. Class number divisibility: The case $l \equiv 1 \pmod 8$.

THEOREM 1. *Let $m = p \equiv 3 \pmod 4$. Then*