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EQUIDISTRIBUTION THEORY IN HIGHER DIMENSIONS

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Let X, Y be complex spaces, and $f: X \rightharpoonup Y$ a meromorphic map. Assume in Y an admissible family $\mathfrak{A} = \{S_b\}_{b \in N}$ of analytic subsets S_b is given. Assume f is almost adapted to $\mathfrak{A}$. The purpose of this paper is to prove that, if f satisfies certain growth conditions, the valence of S_b (for almost all $S_b \in \mathfrak{A}$) grows to infinity at the same rate as the characteristic of f. Here X is assumed to carry an exhaustion function which is, e.g., g-concave, centrally g-convex or g-quasiparabolic.

The results obtained generalize the Casorati-Weierstrass type theorems of Chern [4] [6], Cowen [7], Griffiths-King [12], Stoll [23] [26], Wu [31, II-III] (see also Griffiths [10]).

Introduction. It is well-known that the classical Casorati-Weierstrass theorem is not true in higher dimensions. In fact, the standard example of Fatou-Bierberbach [2, p. 45] gives a holomorphic imbedding of $\boldsymbol{C}^2$ into $\boldsymbol{P}_2$ with a nondense image. Chern [4] first showed that a holomorphic map $f: \boldsymbol{C}^n \to \boldsymbol{P}_n$ whose characteristic grows sufficiently rapidly assumes almost every point in $\boldsymbol{P}_n$. This result was generalized to subvarieties of a general codimension in a complex manifold by Hirshfelder [13] and Stoll [21]-[23]. In Wu [31] certain geometric conditions were given which ensure the Casorati-Weierstrass property. For instance, if $\boldsymbol{C}^n$ is given the Fubini-Study metric, then a nondegenerate quasi-conformal holomorphic map $f: \boldsymbol{C}^n \to \boldsymbol{P}_n$ assumes almost every point in $\boldsymbol{P}_n$. This in fact carries over to a balanced holomorphic map of $\boldsymbol{C}^m$ into $\boldsymbol{P}_n$ (see [10, p. 54]), whose image intersects almost every $(n-p)$-dimensional linear subspace of $\boldsymbol{P}_n$ (where $0 < p \leqq \min(m, n)$).

Let $f: X \rightharpoonup Y$ be a meromorphic map between complex spaces X, Y. Assume in Y an admissible family $\mathfrak{A} = \{S_b\}_{b \in N}$ is given. This means $\mathfrak{A}$ is defined by two holomorphic maps $Y \overset{h}{\leftarrow} M \overset{\pi}{\rightarrow} N$ (where M is a complex space, N a compact complex manifold) such that (i) π is open, surjective; (ii) h is proper, locally trivial at every point of M; (iii) each S_b is the topological image of $\pi^{-1}(b)$ under h, and S_b contains no branch of Y. Then S_b is analytic of pure codimension s in Y for all b. The main purpose of this paper is to establish the equidistribution property that, for almost every $S_b \in \mathfrak{A}$, the valence of S_b grows (over suitable sequence of domains) at the same rate as the characteristic of f. The admissible family defined here is more