PACIFIC JOURNAL OF MATHEMATICS
Vol. 78, No. 2, 1978

REPRESENTATIONS OF WITT GROUPS

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This paper gives a tensor product theorem for the coordinate rings of the finite-dimensional Witt groups. This theorem leads to a demonstration of the equivalence of the representation theory of the Witt groups with that of certain truncated polynomial rings.

Introduction. The Steinberg tensor product theorem [1, Ch. A, §7] for a simply connected, semisimple algebraic group G in characteristic p displays irreducible G-modules as tensor products of Frobenius powers of infinitesimally irreducible G-modules (modules which are irreducible for the kernel G^1 of the Frobenius morphism of G).

A goal of modular representation theory is the expression of the coordinate ring of G in terms of tensor products of Frobenius powers of G-modules which are suitably elementary for G^1. In this paper, we give a tensor product theorem for the finite-dimensional Witt groups. We produce a subcoalgebra C of the coordinate ring A of the m-dimensional Witt group W_m which is isomorphic to the coordinate ring of the kernel W_m^1 of the Frobenius morphism. A is the inductive limit of tensor products of Frobenius powers of C [§3, Theorem].

One can see some things about the representations of W_m. First, every finite-dimensional representation of W_m^1 extends to a representation of W_m on the same representation space [§5]. Second, a representation of W_m on a finite-dimensional vector space V is determined by a family $\{f_1, \cdots, f_n\}$ of commuting endomorphisms of V such that $f_i^{p^m} = 0$. In other words, the representations of W_m on V may be studied via the representations of the algebras $\{k[x_1, \cdots, x_n]/(x_1^{p^m}, \cdots, x_n^{p^m})\}_n$ on V [Theorem, §4]. In particular, the representations of W_m which correspond to the representations of $k[x_1]/(x_1^{p^m})$ give canonical extensions for the representations of W_m^1.

This linear formulation of the representation theory of W_m leaves one with the apparently difficult problem of determining the representation theory of $k[x_1, \cdots, x_n]/(x_1^{p^m}, \cdots, x_n^{p^m})$.

For the definition of the Witt groups, see [2, Ch. 5, §1].

NOTATION. Let A denote the coordinate ring of the m-dimensional Witt group W_m, as a reduced, connected group scheme over the prime field $k = F_p$. For any subcoalgebra C of A which contains k, let $C^{(p^i)}$ be the image of C under the ith-power of the Frobenius