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INVARIANTS OF INTEGRAL REPRESENTATIONS

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Let ZG be the integral group ring of a finite group G. A ZG-lattice is a left G-module with a finite free Z-basis. In order to classify ZG-lattices, one seeks a full set of isomorphism invariants of a ZG-lattice M. Such invariants are obtained here for the special case where G is cyclic of order p^2, where p is prime. This yields a complete classification of the integral representations of G. There are also several results on extensions of lattices, which are of independent interest and apply to more general situations.

Two ZG-lattices M and N are placed in the same genus if their p-adic completions M_p and N_p are Z_pG-isomorphic. One first gives a full set of genus invariants of a ZG-lattice. There is then the remaining problem, considerably more difficult in this case, of finding additional invariants which distinguish the isomorphism classes within a genus. Generally speaking, such additional invariants are some sort of ideal classes. In the present case, these invariants will be a pair of ideal classes in rings of cyclotomic integers, together with two new types of invariants: an element in some factor group of the group of units of some finite ring, and a quadratic residue character (mod p).

For arbitrary finite groups G, the classification of ZG-lattices has been carried out in relatively few cases. The problem has been solved for G of prime order p or dihedral of order $2p$. It was also solved for the case of an elementary abelian (2, 2)-group, and for the alternating group A_4 (see [10a] for references).

The main results of the present article deal with the case where G is cyclic of order p^2, where p is prime. In Theorem 7.3 below, there is a full list of all indecomposable ZG-lattices, up to isomorphism. Theorem 7.8 then gives a full set of invariants for the isomorphism class of a finite direct sum of indecomposable lattices.

Sections 1 and 2 contain preliminary remarks about extensions of lattices over orders. Sections 3 and 5 consider the following problem: given two lattices M and N over some order, find a full set of isomorphism invariants for a direct sum of extensions of lattices in the genus of N by lattices in the genus of M. The results of these sections are applied in §§ 4 and 6 to the special case of ZG-lattices, where G is any cyclic p-group, p prime. Finally, § 7 is devoted to detailed calculations for the case where G is cyclic of order p^2.

Throughout the article, R will denote a Dedekind ring whose