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DISTRIBUTION ESTIMATES OF BARRIER-CROSSING PROBABILITIES OF THE YEH-WIENER PROCESS

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Let $Q = [0, S] \times [0, T]$ be a rectangle and $\{X(s, t): s, t \geqq 0\}$ be the two-parameter Yeh-Wiener process. This paper finds probabilities of $X(s, t)$ crossing barriers of the type $ast + bs + ct + d$ on the boundary ∂Q. These probabilities give lower bounds for the yet unknown probabilities of $X(s, t)$ crossing $ast + bs + ct + d$ on Q. The paper also discusses sharper bounds for the latter probabilities.

1. **Introduction.** Let $\{X(s, t): s, t \geqq 0\}$ be the standard Yeh-Wiener process of two parameters such that it is a separable real Gaussian stochastic process satisfying:

$$X(s, t) = 0 \text{ a.s. if } s \text{ or } t \text{ is } 0 , \tag{1.1}$$

$$\text{the expected value } E\{X(s, t)\} = 0 \text{ at every } s, t \geqq 0 , \tag{1.2}$$

$$E\{X(s, t)X(s', t')\} = \min(s, s')\cdot\min(t, t') . \tag{1.3}$$

Further properties of the process are found in Yeh's [8] and [9].

For the square $D = [0, 1] \times [0, 1]$ and its boundary ∂D, Paranjape and Park [6] showed that the probability

$$P\left\{\sup_{\partial D} X(s, t) \geqq \lambda\right\} = 3N(-\lambda) - e^{4\lambda^2}N(-3\lambda) , \quad \lambda \geqq 0 , \tag{1.4}$$

where $N(\cdot)$ stands for the standard normal distribution function. This probability is a lower bound of the yet unknown probability, $P\{\sup_D X(s, t) \geqq \lambda\}$. It is known (see [4] or [7]) that

$$P\left\{\sup_{D} X(s, t) \geqq \lambda\right\} \leqq 4P\{X(1, 1) \geqq \lambda\} = 4N(-\lambda) . \tag{1.5}$$

Recently Chan [1] showed that, for every $\varepsilon > 0$,

$$P\left\{\sup_{D} X(s, t) \geqq \lambda\right\} \leqq N(\varepsilon)^{-1}P\left\{\sup_{D} X(s, t) \geqq \lambda - \varepsilon\right\} . \tag{1.6}$$

By the same technique as he used in his paper, the upper bound can easily be improved to $N(\varepsilon)^{-1}P\{\sup X(1, t) \geqq \lambda - \varepsilon: 0 \leqq t \leqq 1\} = 2N(-\lambda + \varepsilon)/N(\varepsilon)$. However it turns out to be that even this improved upper bound is not as good as $4N(-\lambda)$ for any $\varepsilon > 0$. In fact

$$4N(-\lambda) < N(\varepsilon)^{-1}P\left\{\sup_{0\leqq t\leqq 1} X(1, t) \geqq \lambda - \varepsilon\right\} , \quad \varepsilon > 0 ,$$