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SUBMANIFOLDS WITH L-FLAT NORMAL CONNECTION OF THE COMPLEX PROJECTIVE SPACE

MASAFUMI OKUMURA

Real submanifolds with L-flat normal connection of the complex projective space are studied. As a special case "a complex submanifold with L-flat normal connection of the complex projective space is necessarily totally geodesic" is proved.

Introduction. As is well known an odd-dimensional sphere S^{2n+1} is a principal circle bundle over a complex projective space $P^n(C)$. The Riemannian structure on $P^n(C)$ is given by the submersion $\pi: S^{2n+1} \to P^n(C)$ which is defined by the Hopf-fibration. If we construct a circle bundle over a real submanifold of $P^n(C)$ in such a way that it is compatible with the Hopf-fibration, the circle bundle is a submanifold of the odd-dimensional sphere. Thus when we want to study submanifolds of the complex projective space it is useful to study the circle bundle over the submanifold. From this point of view, H. B. Lawson, Jr. [2] and the present author [3, 4, 5] have studied real submanifolds of the complex projective space. In the previous paper [5], the author studied relatious between the normal connection of a submanifold of $P^n(C)$ and that of the circle bundle over the submanifold and established the notion of L-flatness for the normal connection of a real submanifold of $P^n(C)$.

The purpose of the present paper is to study submanifolds with L-flat normal connection of $P^n(C)$. The main result is the following.

THEOREM 1. *The totally geodesic complex projective linear subspaces $P^n(C)$ are the only complex submanifolds with L-flat normal connection of $P^{n+p}(C)$.*

In § 1 we state some formulas for real submanifolds of a Kaehlerian manifold and in § 2 we discuss the case when the ambient manifold is the complex projective space. There we explain L-flatness of the normal connection. In § 3, we calculate the Laplacian for a function which is defined on the submanifold and prove some theorems including Theorem 1.

1. Real submanifolds of a Kaehlerian manifold. Let M' be a real $(n + p)$-dimensional Kaehlerian manifold with Kaehlerian