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# THE EVOLUTION OF BOUNDED LINEAR FUNCTIONALS WITH APPLICATION TO INVARIANT MEANS

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**Let $S$ be a topological semigroup and let $X$ be a left translation invariant, left introverted closed subspace of $CB(S)$. Let $m$ and $\bar{\mu}$ be elements of $X^*$, where $\bar{\mu}(f) = \int f d\mu$ for $f$ in $CB(S)$ and $\mu$ is a measure on $S$ which lives on a suitable set. It is shown that the evolution and convolution of $m$ and $\bar{\mu}$ coincide. The same argument carries over to prove that if $X \subset W(S)$, then the evolution and convolution of $m$ and $n$ in $X^*$ are the same (a known result). The topological invariance of invariant means on $X^*$ is discussed.**

**1. Preliminaries.** Let $S$ be a topological semigroup with separately continuous multiplication and $CB(S)$ the Banach space, under supremum norm of bounded real continuous functions on $S$. For each $s$ in $S$, define the left and right translation operators on $CB(S)$ by $(l_s f)(t) = f(st)$ and $(r_s f)(t) = f(ts)$ for all $t$ in $S$, $f$ in $CB(S)$. The subspace $X$ of $CB(S)$ is called left (right) introverted, if for each $m$ in $X^*$ the function $s \to f * m(s) = m(l_s f)(s \to m * f(s) = m(r_s f))$ is in $X$. $W(S)$ denotes the subspace of $CB(S)$ consisting of weakly almost periodic functions, i.e., the functions $f$ such that the set $\{r_s f : s \in S\}$ is conditionally weak compact. $LUC(S)$ ($WLUC(S)$) is the subspace of $CB(S)$ consisting of (weakly) left uniformly continuous functions on $S$, i.e., the functions $f$ such that the map $s \to l_s f$ is norm (weak) continuous. $M_\sigma(S)(M(S))$ denotes the linear space of all real valued signed Baire (regular Borel) measures on $S$. The mapping $T: CB(S) \to M^*(S)$ is the natural embedding of $CB(S)$ into $M^*(S)$ defined by $(Tf)(\mu) = \int f d\mu$ for $f$ in $CB(S)$ and $\mu$ in $M(S)$. Following Granirer [4] $\sigma(CB(S), M_\sigma(S)) = \sigma(C, M_\sigma)$ denotes the weakest topology on $CB(S)$ which makes all linear functionals on $CB(S)$ of type $\int f d\mu$ for $\mu$ in $M_\sigma$ continuous.

For $\mu$ in $M_\sigma(S)$ or in $M(S)$ and $f$ in $CB(S)$ let $\mu * f(t) = \int r_t f d\mu$, $f * \mu(t) = \int l_t f d\mu$ for any $t$ in $S$.

For $\mu$ in $M_\sigma(S)$ or in $M(S)$, $\bar{\mu}$ denotes the functional in $CB^*(S)$ defined by $\bar{\mu}(f) = \int f d\mu$ for $f$ in $CB(S)$.

**2. The main theorem.** Before stating the main theorem we need the following lemma.

LEMMA 2.1. *Let $S$ be a topological semigronp. For $f$ in $CB(S)$*