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ON REPRESENTING ANALYTIC GROUPS WITH THEIR AUTOMORPHISMS

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A real or complex Lie group is said to be *faithfully representable* if it has a faithful finite-dimensional analytic representation. Let G be a real or complex analytic group, and let A denote the group of all analytic automorphisms of G, endowed with its natural structure of a real or complex Lie group. The natural semidirect product $G \rtimes A$ is a real or complex Lie group, sometimes called the *holomorph* of G. We show that if G is faithfully representable and if the maximum nilpotent normal analytic subgroup of G is simply connected then $G \rtimes A$ is faithfully representable.

This result follows quite easily from well-known representation-theoretical results and techniques. What we use is contained in [1, Ch. XVIII], and all the references given below are to this. Nominally, these references cover only the real case. However, as explained loc. cit., both the results and their proofs are almost identical in the complex case.

Thanks are due to Martin Moskowitz who drew my attention to this question and who obtained a number of special results that are consequences of the theorem below and contain suggestions for its proof.

PROPOSITION. *Let G be a faithfully representable real or complex analytic group, and let N be the maximum nilpotent normal analytic subgroup of G. If N is simply connected there is a faithful finite-dimensional analytic representation of G whose restriction to N is unipotent.*

Proof. By Theorem 4.3 (or 4.7), G is a semidirect product $B \rtimes H$, where B is solvable and simply connected, and H is reductive. The construction of B is carried out in the proof of Theorem 4.2, and this shows that, if N is simply connected, one can arrange to have $N \subset B$ (one begins with a semidirect product decomposition of the radical of G having the form $M \rtimes Q$, where Q is reductive, and M is simply connected *and contains* N).

By Theorem 3.1, there exists a faithful finite-dimensional analytic representation ρ of B whose restriction to N is unipotent. Now ρ satisfies the conditions of Theorem 2.2, so that (enlarging the representation space of ρ) one can extend ρ to a finite-dimensional