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NORMAL EXPECTATIONS AND INTEGRAL DECOMPOSITION OF TYPE III VON NEUMANN ALGEBRAS

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Let M be a σ-finite type III von Neumann algebra with separating and cyclic vector ζ (on a not necessarily separable Hilbert space), let C be the center of M, let e be the projection corresponding to the subspace generated by $C\zeta$, and let $\tau(x)$ be the unique element in C with $\tau(x)e=exe$ for x in M. For χ in the spectrum X of C, let ρ_χ be the canonical representation of the state $\tau_\chi(x)=\tau(x)^\frown(\chi)$. The integral $\int\tau_\chi(x)d\nu(\chi)$ induces the central decomposition of M. A separable C^*-algebra B of M is found so that $\rho_\chi(M)''$ has a σ-weakly continuous projection of norm one on $\rho_\chi(B)''$, and $\rho_\chi(B)''$ is a type III factor on an open dense set of X. It is shown that $\rho_\chi(M)''$ is type III and that τ_χ has a decomposition (in the sense of Choquet-Bishop-de Leeuw) as an integral of type III functionals quasi-supported by primary type III functionals for χ in the open dense set.

1. Introduction. One may write every *normal* (i.e., σ-weakly continuous positive linear) functional ϕ of a von Neumann algebra M as an integral of a field of linear functionals over a base space. Several different choices are possible. The field of states (i.e., of positive functionals ψ with $\psi(1) = 1$) can be taken, and the measure can be taken to be a Borel measure *quasi-supported* in the sense of Choquet-Bishop-de Leeuw by the *primary* functionals (i.e., functionals whose canonical representations produce factor von Neumann algebras) in that every Baire set disjoint from the set of primary functionals has measure 0 [26], [37], [44], [45]. It appears there is not much information on whether the measure is supported in some way by functionals whose type (i.e., functionals whose canonical representations have type) corresponds to the type of the algebra M.

One may decompose ϕ in another way. The algebra M may be considered as a Banach module over its center C. The functional ϕ can be written as $(\phi|C)\circ\Phi$ where Φ is in the positive cone $M_\sim^+$ (i.e., $\Phi(M^+)\subset C^+$) of the space $M_\sim$ of σ-weakly continuous C-module homomorphisms of M into C. Defining the field of functionals $\{\phi_\chi|\chi\in X\}$ over the spectrum X of C by $\phi_\chi(x) = \Phi(x)^\frown(\chi)$, one gets the representation

$$\phi(x) = \int\phi_\chi(x)^\frown d\mu(\chi)$$