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REGULAR FPF RINGS

S. PAGE

It is shown that a von Neumann regular ring is FPF (i.e., very faithful finitely generated module is a generator) iff it is self-injective of bounded index.

1. Introduction.

An associative ring R is called a left (F)PF ring if every (finitely generated) faithful module generates the category of left R-modules. Azumaya [1], Osofsky [7], and Utumi [9, 12] characterized the left PF rings as those rings for which any one of the following equivalent conditions holds:

(PF_1) R is left self-injective, semiperfect, and has essential left socle.

(PF_2) R is left self-injective with finitely generated essential left socle.

(PF_3) $R = \bigoplus \sum_{i=1}^{n} Re_i$, $e_i^2 = e_i$ and Re_i is injective with simple essential socle.

(PF_4) R is an injective cogenerator in R-mod.

(PF_5) R is left self-injective and every simple left R-module embeds in R.

C. Faith in [3, 4] has studied semiperfect left FPF rings. In this note we are concerned with von Neumann regular rings which are left FPF. As the conditions PF_1-PF_5 readily point out a von Neumann regular ring which is PF must be semi-simple artinian. In this note we show that if R is von Neumann regular, then R is FPF iff R is of bounded index and left self-injective. It follows that for regular rings left FPF implies right FPF also.

II. Preliminaries.

In what follows R will denote an associative ring with unity and all modules will be unitary left R-modules unless otherwise noted.

A ring R is von Neumann regular if for every $a \in R$ there is an $x \in R$ such that $axa = a$. We will just say R is regular.

DEFINITION. For a set $S \subset M$, M an R-module, let ${}^{\perp}S = \{r \in R: rs = 0 \text{ for all } s \in S\}$. If M is a right R-module, define $S^{\perp} = \{r \in R: sr = 0 \text{ for all } s \in S\}$.

DEFINITION. Let M be an R-module. Let $Z(M)$ be the left singular submodule of M i.e., $Z(M)$ is the set of elements of M whose annihilators are essential left ideals of R. M is called non-singular if $Z(M) = 0$.