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A SPECTRAL SEQUENCE FOR THE HOMOLOGY OF AN INFINITE DELOOPING

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Let $E = \{E_n : n \in Z\}$ be a (-1)-connected infinite loop space: i.e., $\Omega E_{n+1} = E_n$ for all n, and for $n \geqq 0$ the space E_n is $(n-1)$-connected. Then the *stable homology* of E is

$$H_*(E) = \lim_{\longrightarrow} H_{*+n}(E_n)$$

under the suspension homomorphisms. One also has the *unstable homology* $H_*(E_0)$, which with mod p coefficients carries a Pontrjagin product and an action of the mod p Dyer-Lashof algebra R.

It is natural to ask how $H_*(E_0)$ determines $H_*(E)$; and the purpose of this paper is to construct and study the general properties of a spectral sequence whose E^2-term depends functorially on $H_*(E_0)$ as an R-Hopf algebra and whose E^∞-term is the associated graded module of a natural filtration on $H_*(E)$. For simplicity we mainly treat the case $p = 2$.

The spectral sequence developed here is probably identical in the stable range with the iterated barconstruction sequence of D. W. Anderson.[1] We give an entirely different construction, however, which makes evaluation of E^2 easier. In fact, if $\mathscr{A}$ is the category of allowable R-Hopf algebras ([4] I or §2 below) then E^2_s is the sth derived functor of the functor $F_p \otimes_R Q(-)$, where $Q(B)$ is the R-module of indecomposables of $B \in \mathscr{A}$.

The following heuristic description of the resulting E^2-term in favorable cases is due to M. Mahowald. Suppose $\pi_0(E_0)$ is free-Abelian and $H_*(E_0[1])$ is polynomial, where $E_0[1]$ is the connected component of the identity. Then as algebras $H_*(E_0)$ is isomorphic to the homology of a product of ΩS^n's for various n. Now $H_*(\Omega S^n)$ is an Abelian Hopf algebra and hence has formal iterated deloopings defined inductively by

$$B^k H_*(\Omega S^n) = \mathrm{Tor}^{B^{k-1}H_*(\Omega S)^n}(F_2, F_2) .$$

Then our E^1-term is the tensor-product of the corresponding groups $B^\infty H_*(\Omega S^n)$, and the differential d^1 is determined by the Dyer-Lashof action on $H_*(E_0)$.

In §1 of this paper we construct the spectral sequence and outline the algebraic identification of its E^2-term. In §2 we set up the algebraic background required to complete this identification. We

[1] *Added in proof.* W. Dwyer points out that our spectral sequence is *not* identical with Anderson's, since Anderson's E_2-term depends only on the algebra structure of $H_*(E_0)$ while ours depends on the R-module structure as well.