

ALMOST PERIODIC FUNCTIONS ON SEMIDIRECT PRODUCTS OF TRANSFORMATION SEMIGROUPS

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The notion of semidirect product of two transformation semigroups is introduced, and its space of almost periodic functions is expressed as a tensor product. The general techniques developed are applied to the special case of a semidirect product $S \otimes T$ of two semigroups. As a consequence new results are obtained on the characterization of the almost periodic compactification of $S \otimes T$ as a semidirect product of compact semigroups. A related result is the splitting of the enveloping semigroup of a semidirect product of certain flows into a semidirect product of enveloping semigroups.

0. Introduction. Let S and T be semitopological semigroups and $S \otimes T$ a semidirect product of S and T . In an earlier paper [10] we showed that, under certain conditions, the almost periodic (a.p.) compactification $(S \otimes T)'$ of $S \otimes T$ is a semidirect product of the a.p. compactification of T and a certain compact topological semigroup containing a dense homomorphic image of S . A simple corollary of this result is that the space of a.p. functions on $S \otimes T$ is a tensor product of the space of a.p. functions on T and a subspace of a.p. functions on S .

In this paper we introduce the notion of semidirect product of transformation semigroups and determine exactly when its space of a.p. functions may be expressed as a tensor product in analogy with the semigroup case described above. Cast in this general setting the problem of characterizing the space of a.p. functions on a semidirect product of semigroups becomes clear, and the techniques developed lead to elegant necessary and sufficient conditions for $(S \otimes T)'$ to be a semidirect product. As a consequence we are able to show that $(S \otimes T)'$ is a semidirect product for *all* semitopological semigroups S with identity and *all* semitopological groups T , thus generalizing results of [10, 11, 12]. The same conclusion holds if T merely contains a dense subgroup. In a similar vein, but using different techniques, we show that in a wide variety of cases the enveloping semigroup of the semidirect product of two equicontinuous flows is (canonically isomorphic to) a semidirect product of the original enveloping semigroups.