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FIXED POINT THEOREMS IN LOCALLY CONVEX SPACES

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Let C be a convex subset of a nuclear locally convex space that is also an F-space. Suppose $T: C \to C$ is nonexpansive and $\{v_n\}$ is given by the Mann iteration process. It is shown that if $\{v_n\}$ is bounded, T has a fixed point. Also, a sequence $\{y_n\}$ can be constructed such that $y_n \to y$ weakly where $Ty = y$. If C is a linear subspace and T is linear, then $\lim y_n = y$.

1. Introduction. With a few exceptions, the nonnormable locally convex spaces encountered in analysis are nuclear spaces. Precupanu [8]-[11] studied those locally convex spaces whose locally convex spaces whose generating family of seminorms satisfy the parallelogram law, and he called them *H-locally convex* spaces. Precupanu [9] observed that they include all nuclear spaces. This is immediate from Corollary 1, page 102 of [13]. Such a space that is also complete will be called a *generalized Hilbert space*. Theorem 2 generalizes a theorem of Reich [12] which generalizes a result of Dotson and Mann [2]. Reich's ingenious proof is modified to apply in this setting. Theorem 4 generalizes a result of Dotson [1]. His approach to the proof is used, but substantial changes are needed in the details.

Let X be a T_2 locally convex space generated by a family $\{\rho_\alpha: \alpha \in \Delta\}$ of continuous seminorms. The function $\rho: X \to R^\Delta$ is defined by

$$(\rho(x))(\alpha) = \rho_\alpha(x)\ , \qquad x \in X\ , \qquad \alpha \in \Delta\ .$$

ρ satisfies the axioms of norm. The topology t_ρ generated by ρ is the original topology where a t_ρ neighborhood of x is of the form

$$\Omega(x, U) = \{y: \rho(x - y) \in U\}$$

where U is a neighborhood of zero in R^Δ. Thus ρ norms X over R^Δ. A mapping T from X into X is *nonexpansive* if $\rho(Tx - Ty) \leqq \rho(x-y)$ for all $x, y \in X$; that is, $\rho_\alpha(Tx - Ty) \leqq \rho_\alpha(x-y)$ for all $x, y \in X$ and $\alpha \in \Delta$.

We look at the Mann iteration process. Let C be a convex subset of X and suppose T maps C into C. Suppose $A = [a_{nk}]$ is an infinite matrix satisfying:

$$a_{nk} \geqq 0 \quad \text{for all } n \text{ and } k\ ,$$
$$a_{nk} = 0 \quad \text{for} \quad k > n\ ,$$