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ON THE INTERSECTION OF REGRESSIVE SETS

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Let A and B be regressive isols and let $\alpha \in A$ and $\beta \in B$. Let Y be the isol containing the set $\alpha \cap \beta$. We study some basic features of the isol Y, and features of Y in the special case the sum $A+B$ is regressive. We also show that there is a large variety of regressive isols A and B for which the values of Y that are associated, in the above way, are all finite.

1. **Introduction.** The results presented in this paper developed from an interest in sets that are the intersection of two regressive sets, and in the isols that contain such sets. (We shall agree that finite sets are both regressive and retraceable.) The following results are known: the intersection of two retraceable sets is retraceable (this is implicit in [7]), but the intersection of a cosimple retraceable set and a cosimple regressive set need not be regressive [1]. On the other hand ([2]) the intersection of any two regressive sets must, if infinite, have an infinite regressive subset. Several of the facts that are developed here were motivated by the following result from [5], which we state in terms of isols:

THEOREM (*Dekker*). *Let A be a regressive isol and let $\alpha, \beta \in A$. Then $\alpha \cap \beta$ is a regressive set.*

We shall prove one generalization of the above theorem. A part of our result gives the following: if α and β are regressive sets that belong to isols whose sum is regressive, then $\alpha \cap \beta$ is also a regressive set.

For regressive isols A and B, let $A \cap B$ denote the collection of all isols that contain a set which is the intersection of a set in A with a set in B. Various properties of the collection $A \cap B$ are derived in the paper. One, in particular, is the following: given any cosimple regressive isol A, there exists an infinite regressive isol B such that all isols in $A \cap B$ are finite.

We will assume that the reader is familiar with the basic notions in the theory of isols and in the theory of regressive isols. Certain special notions, like *T-retraceability*, are important in the paper, and these will be defined in the sequel. If α is an isolated set then $[\alpha]$ will denote the isol to which α belongs. We recall that sets α and β are *recursively equivalent* if there exists a one-to-one partial recursive function f defined on α and with $f(\alpha) = \beta$.