PACIFIC JOURNAL OF MATHEMATICS
Vol. 79, No. 1, 1978

A REMARK ON INFINITELY NUCLEARLY DIFFERENTIABLE FUNCTIONS

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There is an infinitely nuclearly differentiable function of bounded type from E to R which is not of bounded-compact type, when $E = l_1$, the Banach space of all summable sequences of real numbers.

Let E and F be two real Banach spaces. A mapping $f: E \to F$ is said to be weakly uniformly continuous on bounded subsets of E if for each bounded set $B \subset E$ and each $\varepsilon > 0$, there are $\phi_1, \phi_2, \cdots, \phi_k \in E'$ and $\delta > 0$ such that if $x, y \in B$, $|\phi_i(x) - \phi_i(y)| < \delta (i = 1, 2, \cdots, k)$, then $\|f(x) - f(y)\| < \varepsilon$. $C_w^m(E; F)$ is the space of m-times continuously differentiable mappings $f: E \to F$ satisfying the following conditions:

(1) $\hat{d}^j f(x) \in \mathscr{P}_w({}^jE; F)(x \in E, j \leqq m)$

(2) $\hat{d}^j f: E \to \mathscr{P}_w({}^jE; F)$ is weakly uniformly continuous on bounded subsets of E, where $\mathscr{P}_w({}^mE; F)(m \in N)$ is the Banach space of continuous m-homogeneous polynomials which are weakly uniformly continuous on bounded subsets of E, its norm being the one induced on it by the current norm of $\mathscr{P}({}^mE; F)$. Set

$$C_w^\infty(E; F) = \bigcap_{m=0}^{+\infty} C_w^m(E; F) .$$

$C_w^m(E; F)$ is endowed with the topology τ_b^m generated by the following system of semi-norms

$$f \in C_w^m(E; F) \sup \{\|\hat{d}^j f(x)\|; x \in B, j \leqq m\} ,$$

where B runs through the bounded subsets of E.

For further details we refer to Aron-Prolla [1].

PROPOSITION 1 (Aron-Prolla [1]). *If E' has the bounded approximation property, then $\mathscr{P}_f(E; F)$ is τ_b^m-dense in $C_w^m(E; F)$, for all $m \geqq 1$.*

Hence, since $\|P\| \leqq \|P\|_N$ for every $P \in \mathscr{P}_N({}^mE; F)(m \in N)$, then $\mathscr{E}_{Nbc}(E; F)$ is contained in $C_w^\infty(E; F)$.

PROPOSITION 2 (Aron-Prolla [1]). *Let $f: E \to F$ be a weakly uniformly continuous mapping on bounded sets. If $B \subset E$ is a bounded set, then $f(B)$ is precompact.*

PROPOSITION 3. *$\mathscr{E}_{Nbc}(l_1) \neq \mathscr{E}_{Nb}(l_1)$, that is, there is an infinitely*