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MAPS AND h-NORMAL SPACES

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Further consequences of hard sets are explored in this paper, and some new relations between a space X and its extension δX are shown. A generalization of perfect maps, called δ-perfect maps, is introduced. It is found that among the WZ-maps, these are precisely the ones which pull hard sets back to hard sets. Applications to δX are made. Maps which carry hard sets to closed sets and maps which carry hard sets to hard sets are considered, and it is seen that the image of a realcompact space under a closed map is realcompact if and only if the map carries hard sets to hard sets.

The last part of the paper introduces a generalization of normality, called h-normal, in which disjoint hard sets are completely separated. It is found that X is h-normal whenever υX is normal. The hereditary and productive properties of h-normal spaces are investigated, and the h-normal spaces are characterized in terms of δ-perfect WZ-maps. Finally as an analogue of closed maps on normal spaces, a necessary and sufficient condition is found that the image of an h-normal space under a δ-perfect WZ-map be h-normal.

1. Introduction. All spaces discussed in this paper are assumed Tychonov (completely regular and Hausdorff) and the word *map* means a continuous surjection. The notation of [2] is used throughout. In particular, βX is the Stone-Čech compactification and υX is the Hewitt realcompactification of X.

The following facts concerning hard sets will be used here. They are found in [8] and [9].

DEFINITION 1. For any space X, let $cl_{\beta X}(\upsilon X - X) = K(=K_X)$. A set $H \subseteq X$ is called *hard* (in X) if H is closed as a subset of $X \cup K$. (A characterization of hard sets internal to X is given in [8].) Let δX be the subspace of βX given by $\delta X = \beta X - (K - X)$. Thus $X \subseteq \delta X \subseteq \beta X$.

PROPOSITION 2. *A subset H of space X is hard if and only if there is a compact subset of δX whose restriction to X is H.*

PROPOSITION 3. *Every compact set in X is hard, but every hard set is compact if and only if $X = \delta X$. (Note every pseudocompact space is of this type.)*

PROPOSITION 4. *Every hard set of X is closed, but every closed*