

THE COMMON FIXED POINT THEORY OF SINGLEVALUED MAPPINGS AND MULTIVALUED MAPPINGS

SHIGERU ITOH AND WATARU TAKAHASHI

First, in a locally convex topological vector space, a theorem is proved which extends fixed point theorems by Lau and Fan-Glicksberg. In a strictly convex Banach space, a similar result is obtained, which is a generalization of the fixed point theorem by Bohnenblust-Karlin. In a Banach space which satisfies Opial's condition, a fixed point theorem is given that generalizes both results by Holmes-Lau-Lim and Lami Dozo. In a uniformly convex Banach space, a similar theorem is considered which extends Lim's fixed point theorem. Finally, the existence of common fixed points of a quasi-nonexpansive mapping and a multivalued nonexpansive mapping is established by an elementary constructive method in a Hilbert space. In many cases, preliminary results on nonexpansive or quasi-nonexpansive retractions are obtained which play crucial roles in proving the above theorems.

1. Introduction. De Marr [11] proved that if G is a commutative family of nonexpansive mappings on a compact convex subset K of Banach space, then G has a common fixed point in K . Then results for nonexpansive mappings on weakly compact convex subsets appeared. Browder [5] proved a fixed point theorem for a single nonexpansive mapping on a bounded closed convex subset of a Hilbert space, while Browder [6] and Göhde [19] on a bounded closed convex subset of a uniformly convex Banach space. Kirk [23] obtained a general form of the similar result for a single nonexpansive mapping on a weakly compact convex subset K of a Banach space in the case that K has normal structure. Since then, various fixed point theorems for nonexpansive mappings were given by Belluce and Kirk [2, 3], Takahashi [33, 34], Mitchell [31], Kirk [24], Holmes and Lau [21], Dotson [12], Lau [26], Bruck [9, 10] and Lim [27], etc. Among them, Bruck obtained interesting characterizations of fixed point sets of nonexpansive mappings. There were also Dotson's results [13] on fixed points of quasi-nonexpansive mappings. Lim [28] proved that if K is a weakly compact convex subset of a Banach space and K has normal structure, then K has complete normal structure. Hence combining this with a theorem of Holmes and Lau [21], it follows that if G is a left reversible semigroup of nonexpansive mappings on a weakly compact convex subset K of a