PACIFIC JOURNAL OF MATHEMATICS
Vol. 79, No. 2, 1978

COMPLETE REDUCIBILITY OF ADMISSIBLE REPRESENTATIONS OVER FUNCTION FIELDS

STEPHEN J. HARIS

In his investigations of the "natural domain of validity" for all Siegel Formulae over number fields, Igusa was lead to a certain class of representations, which however, make sense over any field, not just number fields. Calling these representations absolutely admissible, Igusa analyzed their arithmetic nature in "Geometry of absolutely admissible representations" [4], to find the ring of invariants, the stabilizers of various points etc. The objective of [2] and of this paper is to show that for function fields, the absolutely admissible representations arise from the same arithmetic questions concerning the Siegel Formula, as was the case for number fields. In [2] we obtained a list of the composition factors of the representations that arise in this manner. In the present paper we show that these representations are in fact completely reducible, whence for the characteristic of the function field sufficiently large (a bound given explicitly for each group) the arithmetic of invariants discussed in [4] hold for function fields, exactly as for number fields.

The method of proof is cohomological, using the structure theory of semi-simple groups to find a sufficient condition, which will guarantee that the extensions split. A case by case examination shows that this condition is satisfied in every case save for SL_2 and E_6, where further arguments are needed.

The author wishes to thank John Sullivan for helpful conversations concerning the cohomology.

1. A sufficient condition for extensions to split. Let G be a linear algebraic group defined over a field k and fix a universal domain $\Omega \supset k$. Let V, W be two rational G-modules that are finite dimensional vector spaces over Ω and set $W^* = \mathrm{Hom}_\Omega(W, \Omega)$. By the theory of derived functors we can identify $\mathrm{Ext}_G(V, W)$ with $H(G, V \otimes_\Omega W^*)$, since the category of G-modules has enough injectives [3]. In particular $\mathrm{Ext}^1_G(V, W) \cong H^1(G, V \otimes_\Omega W^*)$, whence the extension of W by V splits if $H^1(G, V \otimes_\Omega W^*) = 0$.

PROPOSITION 1. *Let G be a semi-simple algebraic group, V a rational G-module, which is a finite dimensional vector space over Ω. Fix a maximal torus T of G and a Borel subgroup $B \supset T$,*