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FINITE GROUPS WITH A STANDARD SUBGROUP ISOMORPHIC TO $PSU(4, 2)$

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The combined work of M. Aschbacher, G. Seitz, and I. Miyamoto classified finite groups G with a standard subgroup L isomorphic to $PSU(4, 2^n)$ such that either $n>1$ or $C_G(L)$ has noncyclic Sylow 2-subgroups. In this paper, we study the case that $n=1$ and $C_G(L)$ has cyclic Sylow 2-subgroups.

Introduction. A group L is *quasisimple* if L is its own commutator group and, modulo its center, L is simple. A quasisimple subgroup L of a finite group G is *standard* if its centralizer in G has even order, L is normal in the centralizer of every involution centralizing L, and L commutes with none of its conjugates. This definition of standard subgroups is equivalent to the original one given by M. Aschbacher in his fundamental paper [1].

I. Miyamoto has classified [23] finite groups G containing a standard subgroup L isomorphic to $PSU(4, 2^n)$ with $n > 1$ such that $C_G(L)$ has cyclic Sylow 2-subgroups. Part of his argument, however, failed to apply to $PSU(4, 2)$. This exceptional nature of $PSU(4, 2)$ may be explained by the isomorphism

$$PSU(4, 2) \cong PSp(4, 3) \cong P\Omega(5, 3) .$$

Because of this, certain groups of characteristic 3 have standard subgroups isomorphic to $PSU(4, 2)$.

In this paper, we prove the following theorem.

THEOREM. *Let G be a finite group and suppose L is a standard subgroup of G with $L \cong PSU(4, 2)$. Furthermore, assume that $C_G(L)$ has cyclic Sylow 2-subgroups, and let X denote the normal closure of L in G. Then one of the following holds.*

(1) *$X/O(X)$ is a simple group of sectional 2-rank 4.*

(2) *$X \cong PSL(4, 4)$ or $PSU(4, 2) \times PSU(4, 2)$.*

(3) *$N_G(L)/C_G(L) \cong \mathrm{Aut}(L)$, and for each central involution z of L, $C_G(z)$ has a quasisimple subgroup K that satisfies the following conditions:*

(3.1) *$z \in K$ and $W = O_2(K)$ is cyclic of order 4.*

(3.2) *$K/\langle z\rangle$ is a standard subgroup of $C_G(z)/\langle z\rangle$ and W is a Sylow 2-subgroup of $C_G(K/\langle z\rangle)$.*

(3.3) *Either $K/O(K) \cong SU(4, 3)$ or $K/Z(K)$ has a Sylow 2-subgroup isomorphic to a Sylow 2-subgroup of $PSL(6, q)$, $q \equiv 3 \bmod 4$.*