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L^p-ESTIMATES FOR SOLUTIONS TO THE INSTATIONARY NAVIER-STOKES-EQUATIONS IN DIMENSION TWO

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In this paper we derive L^p-estimates for solutions to the instationary nonlinear problem which are known to be valid for solutions to the linear problem. Since the estimates do not depend on t explicitly, they can be used to prove an exponential decay of the solutions if t goes to infinity.

0. **Introduction.** The regularity of the weak solutions[1] to the Navier-Stokes-equations is an outstanding problem in the mathematical theory of fluid dynamics. In the three-dimensional case the answer to this question is still unknown in general, though definite answers have been given in the case of *small* data for *arbitrary large* times, and in the case of *large* data and *small* time intervals, cf. the remarks in [4, Chap. 6].

In the two-dimensional case the problem is much easier to settle: it is well-known that the (unique) weak solution to the Navier-Stokes-equations is regular provided the data are smooth enough. However, the answer is not quite satisfactory since the results of the L^p-theory of the nonstationary hydrodynamic potentials have not been carried over to the Navier-Stokes-equations, e.g., to prove that the solution has square integrable second derivatives one has not only to assume that the external force is square integrable but also that it has a square integrable time derivative.

Recently, v. Wahl filled this gap in proving that in dimension two the solution of the Navier-Stokes-equations has r-summable second derivatives if the right-hand side of the system is r-summable for $2 \leqq r < \infty$. Actually, he gave a detailed proof in the case $r = 2$, and indicated the steps necessary to prove the general result.

The aim of this paper is to give a simple proof of v. Wahl's result. To prove the L^r-estimates for arbitrary $r \geqq 2$ we apply the results of Solonnikov [7, § 17] valid for the *linear* Stokes-equations.

In the interesting case $r = 2$ we shall give an elementary proof relying only on Gronwall's inequality and a well-known interpolation theorem of Nirenberg. In this case we shall obtain an a priori estimate which does not depend on time explicitly. From this result we deduce a number of interesting conclusions concerning the solu-

[1] In the sense of Hopf [2].