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A CHARACTERIZATION OF R^2 BY THE CONCEPT OF MILD CONVEXITY

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Let S be an open, connected set in a locally convex, Hausdorff topological vector space L. If the boundary of S contains exactly one point not a mild convexity point of S and this point is not isolated in $\operatorname{bd} S$, then $\dim L = 2$.

NOTATION. $[S]$ denotes the convex hull of S. $\langle S\rangle$ denotes the interior of $[S]$ relative to the affine closure, aff S, of S. $\operatorname{int} S$, $\operatorname{cl} S$, and $\operatorname{bd} S$ represent the interior, closure and boundary of S, respectively, while $\operatorname{ext} S$ and $\exp S$ denote the sets of extreme and exposed points of S. codim S denotes the codimension of aff S.

DEFINITION. Let S be a set in a topological vector space L. A point x is called a mild convexity point of S if there do not exist two points y and z such that $x \in \langle y, z\rangle$ and $[y, z] \sim \{x\} \subseteqq \operatorname{int} S$. [1].

The proof of Theorem 2 proceeds through some lemmas. Easy proofs are omitted.

LEMMA 1. *A topological vector space over $\boldsymbol{R}$ induces a locally convex, relative topology on every finite-dimensional linear subspace. Hence the relative topology on every finite-dimensional subspace is coarser than the standard Hausdorff topology on the subspace.*

Proof. Suppose the subspace M of L has finite dimension m and U is an arbitrary 0-neighborhood of L. Choose a balanced 0-neighborhood V such that

$$\sum_{1}^{m+1} V \subseteqq U .$$

Then by Caratheodory's theorem [1]

$$V \cap M \subseteqq [V \cap M] \subseteqq U \cap M .$$

LEMMA 2. *Let S be an open set in a topological vector spaces. Suppose $[x, y] \cup [y, z] \subseteqq S$ and $[x, y, z] \cap \operatorname{bd} S$ contains mild convexity points of S only. Then $\langle x, y, z\rangle \subseteqq S$.*

Proof. If x, y, z are collinear then there is nothing to prove; otherwise S intersects aff $\{x, y, z\}$ in a set which is open relative to the standard Hausdorff topology by Lemma 1. Therefore