PACIFIC JOURNAL OF MATHEMATICS
Vol. 79, No. 2, 1978

EXTENDING A BRANCHED COVERING OVER A HANDLE

ALLAN L. EDMONDS

It is shown that if $\varphi: M^n \rightarrow S^n$, $n \geqq 3$, is a branched covering of degree at least 3 and if W^{n+1} is $M^n \times [0, 1]$ with a 2-handle attached, then φ extends to a branched covering $W^{n+1} \rightarrow S^n \times [0, 1]$.

1. Introduction. Let $\varphi: M^n \rightarrow S^n$ be a branched covering, where M^n is a connected n-manifold, $f: \partial B^k \times D^{n-k+1} \rightarrow M^n$ be a flat embedding, and $W^{n+1} = M^n \times [0, 1] \cup_{(f,1)} B^k \times D^{n-k+1}$ be $M^n \times [0, 1]$ with a k-handle attached along $M^n \times 1$ via f. When can one extend φ to a branched covering $\theta: W^{n+1} \rightarrow S^n \times [0, 1]$?

If $k = 1$ and $\deg \varphi \geqq 2$, one always can extend φ [2; (6.1)]. But for $k = 2$ and $\deg \varphi = 2$ one meets obstructions indicated by the fact that the 3-torus T^3 is not a 2-fold branched covering of S^3[4].

In this paper we show (Theorem 4.4) that one can always extend φ if $k = 2$ provided that $\deg \varphi \geqq 3$ and $n \geqq 3$. (For $n = 2$ one would need to assume that $f(\partial B^2)$ does not separate M^2.) The prototype for a result of this sort was proved in a recent paper by J. Montesinos [8] forthe case $n = 3$, when φ is a particular standard 3-fold branched covering of a connected sum of $S^1 \times S^2$'s over S^3.

Again in the case when $k = 3$, $\deg \varphi = 3$, and $n \geqq 4$ one meets further obstructions indicated by the fact that T^4 is not a 3-fold branched covering of S^4 [1].

2. Preliminaries. We shall work in the PL category of piecewise linear manifolds and maps [6]. All embeddings of manifolds in manifolds will be required to be locally flat. The symbols M^n and N^n will denote compact orientable n-manifolds. The symbols B^n and D^n will be reserved for a standard model of a PL n-ball, say $\{x \in \boldsymbol{R}^n: |x_i| \leqq 1, i = 1, \cdots, n\}$, and $S^n = \partial B^{n+1}$ will denote the standard PL n-sphere.

A *branched covering* is a surjective, finite-to-one, open (PL) map $\varphi: M^n \rightarrow N^n$ between n-manifolds. The *singular set* of a branched covering $\varphi: M^n \rightarrow N^n$ is the set of $x \in M^n$ near which φ fails to be a local homeomorphism and is denoted by Σ_φ; the *branch set* of φ is $B_\varphi = \varphi\Sigma_\varphi \subset N^n$.

The *degree* of a branched covering $\varphi: M^n \rightarrow N^n$ is $\deg \varphi = \sup\{\#\varphi^{-1}(y): y \in N^n\}$. One easily verifies that $\deg \varphi$ is the absolute value of the ordinary homological degree of φ as a map.

A *branch homotopy* is a branched covering $\theta: M^n \times [0, 1] \rightarrow N^n \times$