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A SIMPLE MEASURE-PRESERVING TRANSFORMATION WITH TRIVIAL CENTRALIZER

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We show that a certain simple and well-known measure-preserving transformation due to Chacón has the property of commuting only with its powers. We also state a theorem concerning the centralizer of certain rank one transformations. In addition we state without proof the fact that Chacón's example has no factors.

In [5] Ornstein showed that there exists an invertible measure-preserving transformation (automorphism) which is mixing and rank one and, moreover, that any such automorphism has trivial centralizer, i.e., commutes only with its powers. The purpose of this note is to show that a simple and well-known example of a weakly-mixing automorphism, which is due to Chacón [3], also has trivial centralizer (Theorem 1). It is worth observing that Theorem 1 is false for an arbitrary weakly-mixing rank one automorphism. In fact, it is possible to construct a weakly-mixing rank one automorphism S which splits, $S = S_1 \times S_2$.

Although the mixing property of Ornstein's example makes it more remarkable there are at least two reasons why Chacón's example is of some interest in this context. First Ornstein's construction is complicated and nonexplicit in nature, as opposed to Chacón's which could hardly be simpler or more explicit. Secondly, even the examples of weakly-mixing transformations having no roots constructed in [2, 1] are quite difficult compared with the present one which actually does much more.

Chacón's transformation can be most efficiently described as follows. Let T be the shift on $\{0, 1\}^Z$ and define by recursion the finite sequences $\sigma(n)$ of 0's and 1's:

$$\sigma(0) = 0$$
$$\sigma(n) = \sigma(n-1)\sigma(n-1)1\sigma(n-1)\,.$$
$$(\sigma(1) = 0010,\ \sigma(2) = 0010001010010,\ \text{etc.})$$

Let $\bar{\mathscr{O}} \subset \{0, 1\}^Z$ consist of those sequences x such that each finite segment of x is a segment of $\sigma(n)$ for some n. Then it is easy to show that $(\bar{\mathscr{O}}, T)$ is a minimal uniquely ergodic system so there is a unique T-invariant Borel measure μ on $\{0, 1\}^Z$ which is supported on $\bar{\mathscr{O}}$. μ can be defined explicitly on finite sequences x (that is, on cylinder sets) by