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CHARACTERIZATION OF A CLASS OF TORSION FREE GROUPS IN TERMS OF ENDOMORPHISMS

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Characterizations in terms of endomorphisms and quasi-endomorphisms are obtained for torsion free abelian groups with the property that each pure subgroup of finite rank is a quasi-summand. A group has this property if and only if its ring of endomorphisms with finite rank is 2-fold *ct*-transitive, and hence *k*-fold *ct*-transitive for every *k*. This property is equivalent to complete decomposability for countable groups the type set of which satisfies the maximum condition. A stronger version of transitivity is required to describe separable groups the type set of which satisfies the maximum condition; to insure generality, it is shown that the maximum condition does not imply countability of the type set, a result of independent interest.

1. Introduction and preliminaries. All groups considered here are subgroups of a fixed vector space V over the rational number field Q; we shall refer to these torsion free abelian groups simply as "groups". G will always denote a full subgroup of V, i.e., one with torsion quotient V/G. V is thus the divisible hull of G and $r(G) = r(V)$, where r denotes rank. $L(V)$ denotes the algebra of linear transformations of V. $E(G)$ is the endomorphism ring of G and $F(G)$ is the pure ideal of $E(G)$ consisting of all endomorphisms with finite rank. Similarly, $QE(G)$ is the quasi-endomorphism algebra of G and $QF(G)$ is the ideal of elements having finite rank. Familiarity with the concept of quasi-isomorphism is assumed; a complete background may be obtained from [2, 3, 9, 10]. $\dot{\subseteq}$, $\doteq$, $\dot{\cong}$ denote quasi-contained, quasi-equal, and quasi-isomorphic, respectively. We consider $QE(G) = \{f \in L(V): fG \dot{\subseteq} G\}$. Since each element of $E(G)$ induces a unique linear transformation on V, we regard $E(G) \subseteq QE(G)$ and use the same symbol to denote an endomorphism of G and also its induced linear transformation. All sums of groups are direct; e.g., notation such as $G \doteq A + B$ implies that A and B are disjoint groups.

We take the following perspective. For $f \in QE(G)$, define the final rank of f to be the minimum among the cardinal numbers $r(f^n G)$, $n = 1, 2, \cdots$. We assert the

PROPOSITION 1.1. *Each quasi-endomorphism of G with finite positive final rank (especially any such endomorphism of G) induces a quasi-decomposition of G.*