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ADDENDUM TO "FIXED POINTS OF AUTOMORPHISMS OF COMPACT LIE GROUPS"

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THEOREM. Let G be a compact, connected Lie group and let h be an endomorphism of G. Then the rank of the Lie group $\Phi(h)$ is equal to the dimension of the graded vector space $\Phi(Ph^*)$.

The statement of the main result (Theorem 1.1) of [1] is unnecessarily restrictive. The result is stated only for automorphisms, but the theorem is in fact true for all endomorphisms.

The proof is the same as in [1] except for the following, which replaces the argument on pages 82-83. Let $\eta = dh: \mathfrak{G} \to \mathfrak{G}$ be the differential of h, where $\mathfrak{G}$ is the Lie algebra of G. Then η induces $Q\eta_*: QH_*(\mathfrak{G}) \to QH_*(\mathfrak{G})$ on the indecomposables in the homology of $\mathfrak{G}$. By de Rham's theorem (see [2]) and Proposition 3.10 of [3], it is sufficient to prove

$$(*) \qquad \operatorname{rank} \Phi(\eta) = \dim \Phi(Q\eta_*) .$$

For the same reasons, we already know that (*) is true if $\mathfrak{G}$ is abelian (Propositions 2.2 and 2.3 of [1]) or if $\mathfrak{G}$ is semisimple and η is an automorphism (Lemma 3.1). Write $\mathfrak{G} \cong \mathfrak{Z} \oplus \mathscr{D}\mathfrak{G}$ where $\mathfrak{Z}$ is the center of $\mathfrak{G}$ and $\mathscr{D}\mathfrak{G}$ is semisimple. Then write $\mathfrak{Z} \cong \mathfrak{Z}_a \oplus \mathfrak{Z}_b$ where $\mathfrak{Z}_a = \eta^{-1}(\mathfrak{Z})$. Let $\eta_a: \mathfrak{Z}_a \to \mathfrak{G}$ be the restriction of η to $\mathfrak{Z}_a$.

For $p: \mathfrak{G} \to \mathfrak{Z}_a$ the projection, $p\eta_a$ is an endomorphism of an abelian Lie algebra. So (*) is true for $p\eta_a$ - and therefore for η_a. Since $\mathfrak{Z}_b \cap \eta(\mathfrak{Z}_b) = 0$, we conclude that $\dim \Phi(Q\eta_{b*}) = 0$. Let $\mathscr{D}\eta: \mathscr{D}\mathfrak{G} \to \mathscr{D}\mathfrak{G}$ be the restriction of η. We can write $\mathscr{D}\mathfrak{G} \cong \mathfrak{A}_1 \oplus \cdots \oplus \mathfrak{A}_N \oplus \mathfrak{B}$ where the restriction η_i of $\mathscr{D}\eta$ to each $\mathfrak{A}_i$ is an automorphism and the behavior of $\mathscr{D}\eta$ on the fixed points is determined by the η_i. Since (*) is true for each η_i, it holds for $\mathscr{D}\eta$ as well. Finally,

$$\operatorname{rank} \Phi(\eta) = \operatorname{rank} \Phi(\eta_a) + \operatorname{rank} \Phi(\mathscr{D}\eta)$$

which completes the proof.

I thank the referee for correcting an error in an earlier version of this paper.

REFERENCES

1. R. Brown, *Fixed points of automorphisms of compact Lie groups*, Pacific J. Math., **63** (1976), 79-87.