

UNIVERSAL DERIVATIONS AND UNIVERSAL RING CONSTRUCTIONS

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If a K -ring S is constructed from a K -ring R by adjoining certain new generators and relations, then the S -bimodule $\Omega_K(S)$ with a universal K -derivation $d: S \rightarrow \Omega_K(S)$ can be constructed from the corresponding R -bimodule $\Omega_K(R)$ by extending scalars to S , and adjoining formal derivatives of the new generators and relations. By studying this bimodule it is shown that a large number of natural universal constructions preserve the class of right hereditary K -rings (K semi-simple Artinian), including the constructions of universal localization (which had resisted earlier techniques) and certain direct limits of known constructions. The same technique gives information on Euler characteristics of modules (Lewin-Schreier formulas). To study universal localizations of a ring R which may not contain a semisimple Artin ring K , a different technique is used.

Let R be a ring (associative, with 1) and $f: P \rightarrow Q$ a homomorphism of finitely generated projective right R -modules. Then there exists an R -ring $S = R \langle f^{-1} \rangle$ having a map of projectives, $f^{-1}: Q \otimes_R S \rightarrow P \otimes_R S$ inverse to f (more precisely, inverse to $f \otimes_R S$), and such that S and f^{-1} are *universal* for this property. This construction generalizes that of adjoining to R a universal inverse of an *element* $f \in R$. We may simultaneously adjoin such inverses to a whole family of maps. We shall call an R -ring S so obtained a *universal (2-sided) localization* of R . The main result of this paper—of which we shall prove two versions by two different methods—is:

- (0) A universal localization S of a right hereditary ring R is right hereditary.

Universal localization was one of a large class of constructions examined in [2] and [3], including also the formation of the coproduct S of a family of rings R_α , the universal adjunction to a ring R of an isomorphism between two given finitely generated projective R -modules P and Q , and the adjunction to R of a universal idempotent endomorphism of a finitely generated projective module P . For these last three and many others, it was proved that when the construction is performed “over” a semisimple Artinian base ring K , a large part of the module theory of the constructed