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FUNCTIONS WHICH OPERATE ON THE REAL PART OF A UNIFORM ALGEBRA

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Three theorems are proved to the effect that a nonaffine function h on an interval cannot operate by composition on the real part of a uniform algebra on X unless the algebra is $C(X)$. The additional hypotheses necessary are, respectively, that h be continuously differentiable, that h be "highly" nonaffine in a suitable sense, and that h operate in a rather weakly bounded manner. These results contain and extend work of J. Wermer and of A. Bernard.

1. **Introduction.** Given a space of functions, its symbolic calculus is a standard object of study, particularly if the space is associated with a Banach algebra. This paper is concerned with the space $\mathrm{Re}A = \{\mathrm{Re}(f)\colon f \in A\}$ where A is a uniform algebra on a compact Hausdorff space X. It is an old conjecture that, unless $A = C(X)$, the symbolic calculus of $\mathrm{Re}A$ is trivial in that the only functions which operate by composition on $\mathrm{Re}A$ are the affine functions $t \to at + b$, which obviously operate.

Precisely, suppose A is a uniformly closed subalgebra of $C(X)$ which contains the constant functions and separates the points of X, I is an interval, and $h\colon I \to \boldsymbol{R}$ is not the restriction of an affine function. The conjecture is that under these cooditions, if h operates by composition on $\mathrm{Re}A$ in the sense that $h \circ u \in \mathrm{Re}A$ whenever $u \in \mathrm{Re}A$ has range in I, then it follows that $A = C(X)$. We shall prove three theorems along these lines.

The history of this problem probably begins with J. Wermer's paper [6], whose conclusion that $\mathrm{Re}A$ cannot be closed under products is equivalent to the conjecture for $h(t) = t^2$ (and, by induction on degree, implies the conjecture for any polynomial of degree at least 2) on any interval. Some time later, A. Bernard [2] proved the conjecture for $h(t) = |t|$ on $I = \boldsymbol{R}$. Our first two results, like Wermer's and Bernard's, place restrictions on h. Either contains Wermer's theorem, but not Bernard's.

THEOREM 1. *Suppose I is an open interval and $h\colon I \to \boldsymbol{R}$ is not affine but is continuously differentiable. If h operates by composition on* $\mathrm{Re}A$, *then* $A = C(X)$.

THEOREM 2. *Suppose that $h\colon I \to \boldsymbol{R}$ and that I contains a nondegenerate subinterval J such that h is not affine on any nonde-*