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OSCILLATION RESULTS FOR A NONHOMOGENEOUS EQUATION

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The purpose of this note is to investigate oscillatory properties of solutions of the equation

$$y''+p(t)y=f(t) \tag{1}$$

via the transformation $y(t)=u(t)z(t)$ where $u(t)$ is a solution of the equation

$$u''+p(t)u=0\ . \tag{2}$$

Equation (2) is assumed to be nonoscillatory throughout the paper. This represents a distinct change from most of the recent work concerning oscillation in equation (1).

The transformation $y(t) = \phi(t)z(t)$ transforms equation (1) into

$$(\phi^2z')' + \phi(t)(\phi''(t) + p(t)\phi(t))z = f(t)\phi(t)\ . \tag{3}$$

If $\phi(t)$ is a solution of (2) then (3) becomes

$$(\phi^2z')' = f(t)\phi(t)\ . \tag{3'}$$

Equation (3′) enables us to characterize the oscillatory behavior of solutions of (1) in terms of the forcing function $f(t)$ and the nonoscillatory solutions of equation (2). The need for "explicit" sign conditions on $p(t)$ is eliminated. However, some implicit sign conditions will be assumed, that is, the solution $\phi(t)$ of equation (2) will be given properties that are implied by specific sign conditions on $p(t)$.

In recent articles Macki [10] and Komkov [7] have pointed out the usefulness of the transformation $u(t) = \phi(t)z(t)$ in studying qualitative properties of the differential equation

$$(r(t)u')' + p(t)u = 0\ .$$

As usual a nontrivial solution $y(t)(u(t))$ of equation (1) [resp. (2)] is oscillatory if on each ray $(a, \infty)(a > 0)$ there exists a $t_0 \in (a, \infty)$ with $y(t_0) = 0$ $(u(t_0) = 0)$. Equation (1) [resp. (2)] is oscillatory if all solutions are oscillatory. A solution $y(t)$ [resp. $u(t)$] of equation (1) [resp. (2)] is nonoscillatory if it is eventually nonzero. It is well known that all solutions of equation (2) are either oscillatory or nonoscillatory. The functions $p(t)$ and $f(t)$ are assumed to be continuous on $[0, \infty)$, so only solutions on the interval $[0, \infty)$ will be