PACIFIC JOURNAL OF MATHEMATICS
Vol. 80, No. 1, 1979

# SEMIGROUPS OF CONTINUOUS TRANSFORMATIONS AND GENERATING INVERSE LIMIT SEQUENCES

G. EDGAR PARKER

**Suppose that $T$ denotes a strongly continuous semigroup of continuous transformations on a closed subset $C$ of a complete metric space. For arbitrary decreasing sequences $\{\delta_n\}_{n=1}^{\infty}$ and $\{\alpha_n\}_{n=1}^{\infty}$ of positive numbers converging to 0, the inverse limit spaces generated by $\{T(\delta_n)(C), T(\delta_n - \delta_{n+1})\}_{n=1}^{\infty}$ and $\{T(\alpha_n)(C), T(\alpha_n - \alpha_{n+1})\}_{n=1}^{\infty}$ are homeomorphic and contain a dense one-to-one continuous image of $C$. Conversely, given an inverse limit system with bonding maps $\{f_n\}_{n=1}^{\infty}$ so that (i) $f_n: C \to C$, (ii) if $x$ is in $C$, $\lim_{n\to\infty} f_n(x) = x$, and (iii) $f_{n+1} \circ f_{n+1} = f_n$, conditions are given under which a semigroup,and consequently a family of homeomorphic inverse limits, can be recovered.**

**Examples are given which illustrate analytical applications and topological implications.**

## 1. Introduction.

This paper deals with the generation of strongly continuous semigroups of continuous transformations. Historically the notion of generation of a strongly continuous semigroups has been the identification of it with the differential equation its trajectories satisfy. Work in [2] shows that for semigroups of nonlinear transformations, this association is not always possible. Recent work by Kobayashi [4], Kobayasi [5], and the author [6] has shown that under various conditions, given the existence of a strongly continuous semigroup, approximating semigroups which must be associated with a differential equation can be constructed. The purpose of this paper is to show that the problem of initial construction of a semigroup (thus establishing the hypothesis in the work mentioned above) is that of constructing a special sequence of functions which in turn generates a family of homeomorphic inverse limit sequences.

Theorem 1 establishes a basic topological structure inherent in semigroups and relates that structure to the set of initial conditions. Theorem 2 is a partial converse to Theorem 1. From six conditions (conditions (i) and (ii) reflect the inverse limit structure demonstrated in Theorem 1; condition (iii) reflects the algebraic structure of semigroups; conditions (iv)-(vi) are not present in all semigroups but do occur in many examples which have been studied) a strongly continuous semigroup on $[0, \infty)$ is recovered from a sequence of functions. Four lemmas provide the proof of Theorem 2.

## 2. Definitions and theorems.