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ON 2-DIMENSIONAL CW-COMPLEXES WITH A SINGLE 2-CELL

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In this paper we are interested in finite connected 2-dimensional CW-complexes, each with a single 2-cell. We show any two such complexes have the same homotopy type if their fundamental groups are isomorphic. In fact, there is a homotopy equivalence inducing any isomorphism of the fundamental groups. We also study the homotopy factorizations of such spaces into finite sums.

In this paper we are interested in finite connected 2-dimensional CW-complexes with a single 2-cell. Each such CW-complex has the homotopy type of the cellular model $C(\mathscr{R})$ of some finite one-relator presentation

$$\mathscr{R} = (x_1, \cdots, x_n: R)$$

of $\Xi = \pi_1 X$. If the single relator R is not a proper power, it is known that the cellular model $C(\mathscr{R})$ is aspherical (see [10], [1], or [4]), hence it is determined up to homotopy type by its fundamental group. If the single relator R is a proper power, $C(\mathscr{R})$ is not aspherical, nevertheless we are able to prove the following:

THEOREM 1. *Any two finite connected 2-dimensional CW-complexes, each with a single 2-cell, have the same homotopy type if their fundamental groups are isomorphic. In fact there is a homotopy equivalence inducing any isomorphism of the fundamental groups.*

Our proof makes use of Lyndon's resolution for one-relator groups [10] and some combinatorial results on one-relator groups which can be found in the book by Magnus, Karass, and Solitar [11].

Theorem 1 has these corollaries:

COROLLARY 1. *Let X and Y be two finite connected 2-dimensional CW-complexes, each with a single 2-cell. Then $X \simeq Y$ if $X \vee L \simeq Y \vee M$ where L and M are finite CW-complexes with isomorphic fundamental groups. Thus $X \simeq Y$ if and only if $X \vee L \simeq Y \vee L$ where L is any finite CW-complex.*

Proof. We have $\pi_1 X * \pi_1 L \approx \pi_1 Y * \pi_1 M$. Because all groups involved are finite generated, we can write these as free product of