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ON SINGULAR INDICES OF ROTATION FREE DENSITIES

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The properties of singular indices of nonnegative rotation free densities on $\Omega(\lambda)=\{\lambda<|z|\leqq 1\}(\lambda\geqq 0)$ will be studied. The relation between the singular index $\alpha(P)$ of a nonnegative rotation free density P on $\Omega(\lambda)(\lambda>0)$ and the Martin compactification $\Omega(\lambda)^*_P$ of $\Omega(\lambda)$ with respect to the elliptic equation $\Delta u=Pu$ will be established.

A nonnegative locally Hölder continuous function $P(|z|)(|z|=r)$ on $\Omega(\lambda)$ is called *a nonnegative rotation free density* on $\Omega(\lambda)$. The *singular index* $\alpha(P)$ of a density P at $r=\lambda$ is the quantity given by

$$\alpha(P)=\lim_{r\to\lambda}e_1(r)/e_0(r)\ ,$$

where $e_j(r)(j=1,2)$ is a unique bounded solution of the equation

$$\frac{d^2}{dr^2}e(r)+\frac{1}{r}\frac{d}{dr}e(r)-(P(r)+j^2/r^2)e(r)=0\ (j=0,1)$$

on $(\lambda, 1]$ with $e_j(1)=1$ for $\lambda=0$, and furthermore with $\lim_{r\to\lambda}e_j(r)=0$ for $\lambda>0$. In particular, $\alpha(0)$ with $P\equiv 0$ on $\Omega(\lambda)(\lambda\geqq 0)$ will be referred to as the *harmonic index* at $r=\lambda$. The *elliptic dimension,* including the harmonic dimension with $P\equiv 0$, of a density P at the ideal boundary $|z|=\lambda$, $\dim\mathscr{P}(\lambda)$ in notation, is the dimension of the half module $\mathscr{P}(\lambda)$ of the positive solutions u of $\Delta u=Pu$ on $\Omega(\lambda)$ with $u=0$ on $|z|=1$. The elliptic dimension is an ideal boundary property (M. Heins [2], K. Hayashi [1], and M. Ozawa [12], [13]). The *Picard principle* is said to be valid for a density P at $|z|=\lambda$ if $\dim\mathscr{P}(\lambda)=1$ or equivalently $\alpha(P)=0$ (M. Nakai [8]).

In [8] M. Nakai had shown that *the singular index $\alpha(P)$ at $\lambda=0$ determines the Martin compactification in the following ways*:

$$\Omega(0)^*_P\approx\{\alpha(P)\leqq|z|\leqq 1\}\ ,\quad \alpha(P)=0 \text{ or } \alpha(P)>0$$

in the sense of homeomorphism for any nonnegative rotation free density P on $\Omega(0)$ and each ideal boundary point is minimal.

It will be shown that *the singular indices are linearly ordered in the sense that $\alpha(P)\leqq\alpha(P_1)$* if $P\leqq P_1$ on $\Omega(\lambda)(\lambda\geqq 0)$. In particular, *the harmonic index which is equal to $(-2\lambda\log\lambda)(1-\lambda^2)^{-1}$* for $\lambda>0$ *minimizes the singular indices at $r=\lambda$ among the nonnegative rotation free densities on $\Omega(\lambda)$.* As a counterpart of