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CHARACTERS OF AVERAGED DISCRETE SERIES ON SEMISIMPLE REAL LIE GROUPS

REBECCA A. HERB

Let G be a real simple Lie group of classical type having a compact Cartan subgroup. Then G has discrete series representations. The purpose of this paper is to establish explicit formulas for certain sums of discrete series characters. These "averaged" discrete series characters have simple formulas which can be used for certain problems in harmonic analysis on G, for example, for the computation of the Plancherel measure on $\hat{G}$.

1. Introduction. Let G be a connected, acceptable, semisimple real Lie group with finite center. Suppose that G has a compact Cartan subgroup T. Then G has discrete series representations. The characters of these representations were initially described by Harish-Chandra in [2]. The characters have simple formulas on T. On the noncompact Cartan subgroups, the formulas are complicated, and contain certain integer constants which Harish-Chandra did not compute.

Using the procedure described in [2], these constants can be computed if related constants are known for each type of simple root system which is spanned by a strongly orthogonal set of roots. These are the root systems of types A_1, B_n, C_n, $D_{2n} (n \geqq 2)$, E_7, E_8, F_4, and G_2, and they correspond to the complex simple Lie groups for which the split real form has a compact Cartan subgroup, and hence discrete series representations. Partial solutions to the problem of computing these constants have been given in [4, 5, 6, 7, 8, 10, 11, 12]. A complete solution is now available in work of T. Hirai [11]. Hirai's formulas express discrete series constants for groups of arbitrary rank in terms of constants for groups of real rank one and two.

Explicit formulas for discrete series characters, besides being of interest for the representation theory of G, are needed for harmonic analysis on G. However for some of these problems, for example, computation of the Plancherel measure on $\hat{G}$, it is necessary only to have certain sums of discrete series characters.

Let $\mathfrak{g}$ and $\mathfrak{t}$ denote the Lie algebras of G and T respectively, and $\mathfrak{g}_C$ and $\mathfrak{t}_C$ their complexifications. Then the discrete series characters of G are parameterized by regular elements τ in a lattice $L_T \subseteqq \sqrt{-1}\mathfrak{t}^*$. The Weyl group W of the pair $(\mathfrak{g}_C, \mathfrak{t}_C)$ acts on L_T. Instead of the characters $(-1)^q \varepsilon(\tau)\theta_\tau$ defined by Harish-Chandra in [2], we