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FOURIER SERIES WITH BOUNDED CONVOLUTION POWERS

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Let f be defined on T^n and have an absolutely convergent Fourier series

$$f(e^{i\sigma})=\Sigma f_m e^{im\cdot\sigma} .$$

Set $||f||=\Sigma|f_m|$. In this paper sufficient conditions for $||f^k||=0(1)$, as $k\to\infty$, are obtained.

THEOREM. *Let f be defined on T^n, have an absolutely convergent Fourier series and satisfy*

$$(1)\qquad |f(e^{i\sigma})| \leqq 1 \text{ for all } \sigma .$$

If for each σ such that $|f(e^{i\sigma})| = 1$ there exists a rotation λ of R^n, a polynomial ρ such that $\mathrm{Re}\,\rho(\tau) > 0$ *for all $\tau \neq \vec{0}$, an n-tuple p of positive integers such that $\rho(\langle r^{1/p_i}\tau_i\rangle) = r\rho(\tau)$ for all $r > 0$, and a function γ in $C^m(R^n)$, $m = \max(n+1, p_1, p_2, \cdots, p_n)$, such that*

$$(2)\qquad \gamma(\tau) = o\left(\sum_{i=1}^{n} \tau_i^{p_i}\right), \quad \tau \longrightarrow \vec{0} ,$$

and if for all τ in some R^n-neighborhood of $\vec{0}$

$$(3)\qquad f(e^{i(\sigma+\lambda(\tau))}) = c\exp(\beta\cdot\tau i - (\rho+\gamma)(\tau)) ,$$

where $|c| = 1$, $\beta \in R^n$, then

$$(4)\qquad ||f^k|| = 0(1), \text{ as } k \longrightarrow \infty .$$

It is shown in § 2 that this theorem extends a result obtained by B. M. Schreiber in 1970.

1. Introduction. Let B and D denote the open and closed unit discs respectively. The problem of characterizing those functions f of n complex variables which are analytic on D^n and for which (4) holds has been solved only for $n = 1$. See [1, 2]. For the general case, n arbitrary, sufficient conditions on f and necessary conditions on f have been given [9, 6] and related problems have been studied [4, 5]. Unsolved, even in the case $n = 1$, is the problem of determining all functions analytic on B^n for which (4) holds. This problem is equivalent to the problem of determining all endomorphisms of the Banach algebra of power series of n complex