PACIFIC JOURNAL OF MATHEMATICS
Vol. 80, No. 1, 1979

THREE-DIMENSIONAL OPEN BOOKS CONSTRUCTED FROM THE IDENTITY MAP

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Three-dimensional manifolds are constructed as open books, using the identity diffeomorphism. The open book constructed in this way with (non)orientable page of Euler characteristic χ is the connected sum of $(1-\chi)$ copies of the (non)orientable S^2 bundle over S^1

Introduction. We investigate orientable and nonorientable three-dimensional manifolds which are open books according to the following definition of Winkelnkemper [2].

DEFINITION. A manifold of dimension n is said to have an open book description if it can be constructed using a co-dimension 2 submanifold ∂V and a diffeomorphism $h: V \to V$ of an $(n-1)$-dimensional manifold with boundary ∂V. h is required to be the identity map in a neighborhood of ∂V. The construction is to form the mapping torus $(V \times I)/(v, 0) = (h(v), 1)$ and then to identify $(v, t) = (v, t')$ for all v in ∂V and t, t' in I. The image of the copies of ∂V in the resulting manifold is called the binding of the open book and the circle's worth of copies of V are called the pages.

Related results appear in the recent book of Rolfsen [1].

Statement of results.

THEOREM 1. *If $V = S_g - n\mathring{B}^2$, the surface of genus g with n disjoint, open discs removed from it, then the open book produced by setting h equal to the identity map is the connected sum of $(2g + (n-1))$ copies of $(S^1 \times S^2)$. (Adopt the convention that zero copies of $(S^1 \times S^2)$ will refer to S^3.)*

THEOREM 2. *If $V = P_k - n\mathring{B}^2$, the 2-sphere with k cross-caps attached and n disjoint, open discs removed from it, then the open book produced by setting h equal to the identity map is the connected sum of $(k + (n-1))$ copies of the Klein bottle of dimension three. $(k \geqq 1, n \geqq 1)$*

By the three-dimensional Klein bottle we mean the nonorientable S^2 bundle over S^1, $(S^2 \times I)/(x, y, z, 0) = (-x, y, z, 1)$.

Proofs of results.