PACIFIC JOURNAL OF MATHEMATICS
Vol. 80, No. 1, 1979

THE IWASAWA INVARIANT μ FOR QUADRATIC FIELDS

FRANK GERTH III

We let k_0 be a quadratic extension field of the rational numbers, and we let l be a rational prime number. In this paper we show that there exists a constant c (depending on k_0 and l) such that the Iwasawa invariant $\mu(K/k_0) \leqq c$ for all Z_l-extensions K of k_0. In certain cases we give explicit values for c.

1. Introduction. We let Q denote the field of rational numbers, and we let l denote a rational prime number. We let k_0 be a finite extension field of Q, and we let K be a Z_l-extension of k_0 (that is, K/k_0 is a Galois extension whose Galois group is isomorphic to the additive group of the l-adic integers Z_l). We denote the intermediate fields by $k_0 \subset k_1 \subset k_2 \subset \cdots \subset k_n \subset \cdots \subset K$, where Gal (k_n/k_0) is a cyclic group of order l^n. We let A_n denote the l-class group of k_n (that is, the Sylow l-subgroup of the ideal class group of k_n). In [5, §4.2], Iwasawa proves that $|A_n| = l^{e_n}$, where

$$e_n = \mu l^n + \lambda n + \nu \tag{1}$$

for n sufficiently large, and μ, λ, ν are rational integers (called the Iwasawa invariants of K/k_0) which are independent of n. Also $\mu \geqq 0$ and $\lambda \geqq 0$.

Next we let W be the set of all Z_l-extensions of k_0. If $K \in W$, we define

$$W(K, n) = \{K' \in W \mid [K \cap K' : k_0] \geqq l^n\} .$$

Thus $W(K, n)$ consists of all Z_l-extensions of k_0 that contain k_n, where k_n is the unique subfield of K such that $[k_n: k_0] = l^n$. We topologize W by letting $\{W(K, n)$ for $n = 1, 2, \cdots\}$ be a neighborhood basis for each $K \in W$. It can be proved that W is compact with this topology (see [4, §3]). Next we let W' be the set of Z_l-extensions of k_0 with only finitely many primes lying over l. In [4, Proposition 3 and Theorem 4], Greenberg proves that W' is an open dense subset of W and that the Iwasawa invariant μ is locally bounded on W'. So if $K \in W'$, there exists an integer n_0 and a constant c depending only on K such that $\mu(K'/k_0) < c$ for all Z_l-extensions K' of k_0 with $[K \cap K': k_0] \geqq l^{n_0}$. Greenberg suggests that perhaps μ is bounded on W; that is, perhaps there exists a constant c such that $\mu(K'/k_0) < c$ for every $K' \in W$. If there is only one prime of k_0 above l, then Greenberg does prove in [4, Theorem 6] that μ is bounded on W.

In this paper we shall prove that μ is bounded on W if k_0 is a