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# CANCELLING 1-HANDLES AND SOME TOPOLOGICAL IMBEDDINGS

MICHAEL H. FREEDMAN

**In this note we use the existence of a certain type of handle decomposition (see corollary) for compact simply connected P. L. 4-manifolds and R. Edwards results on the double suspension conjecture to prove:**

THEOREM 2. *Let* $\alpha \in H_2(M; Z)$ *where* $M$ *is a compact simply connected* P. L. 4-*manifold. Then there is a proper topological imbedding* (*possible nonlocally flat*) $\theta: S^2 \times R \rightarrow M \times R$ (*mapping ends to ends*) *with* $\theta_*[S^2 \times R] = \bar{\alpha} \in H_2\ (M \times R; Z)$. $\bar{\alpha}$ *is the image of* $\alpha$ *under* $\times R$. *Proper, here, means inverse images of compact sets are compact.*

In [2], we considered the problem of constructing smooth proper imbeddings, $\theta$, and showed that if $\alpha$ is characteristic (dual to $w_2(\tau(M))$, the only obstruction to the existence of $\theta$ is an Arf invariant which is equal to the Milnor-Kervaire number ($=$(signature $(M) - \alpha \cdot \alpha/8$)(mod 2)) when $M$ is closed and that if $\alpha$ is ordinary (not dual to $W_2(\tau(M))$ there is no obstruction. This suggests two problems: (1) Can $\theta$ always be arranged to be topologically locally flat, and (2) can $\theta$ always be arranged to be P. L.?

Here is our "handle cancellation" theorem:

THEOREM 1. *Let* $M$ *be any compact connected* P. L. *manifold of dimension* $= m$ (*assume* $M$ *orientable if* $m = 3$). *Let* $N$ *be a compact connected codimension* 0 *submanifold of* $\partial M$. *If* $\pi_1(M, N) = 0$, *then there is a codimension* 0 *submanifold*, $\bar{N}$, *of* $M$ *with*: (1) $N \hookrightarrow \bar{N}$, (2) *the inclusion* $N \hookrightarrow \bar{N}$ *is a homotopy equivalence*, (3) $M = \bar{N} \cup$ 2-*handles* $\cup$ 3-*handles* $\cup \cdots \cup$ $m$-*handles*.

*Note*. The P. L. category is convenient here since handle decompositions always exists.

*Proof*. If $n \geqq 5$, the usual arguments for cancelling handles produce the desired $\bar{N} \overset{\text{P. L.}}{=\!=} N \times I$ (see Appendix [3]). We need only consider the cases $m = 3$ or 4.

Let $m = 4$ and let $\mathscr{H}(M, N)$ be a handle decomposition of $M$ relative to $N$. We may assume $\mathscr{H}(M, N)$ has no zero-handles.

Let $\{h_i^1\} = \{D_i^1 \times D_i^{m-1}\}$ be the 1-handles. Let $\{c_i\}$ be closed curves