PACIFIC JOURNAL OF MATHEMATICS
Vol. 80, No. 1, 1979

THE FUNDAMENTAL DIVISOR OF NORMAL DOUBLE POINTS OF SURFACES

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Let W be a surface with a normal singular point w. Consider the minimal resolution of that singularity, $\pi\colon W' \to W$. Let $\pi^{-1}(w) = Y = Y_1 \cdots Y_d$, where the Y_i are distinct irreducible curves on W'. We are interested in two divisors on W' both of which have support on Y. These divisors are Z, the fundamental divisor, and M, the divisor of the maximal ideal. In general $Z \leqq M$. In this thesis we show that if w is a double point singularity which satisfies certain conditions, then $Z = M$.

Introduction. Let A denote a normal, two-dimensional local ring. For simplicity assume that the residue field, k, of A is algebraically closed. Let $\pi\colon Y \to \mathrm{Spec}\,(A)$ be a birational proper map with Y regular, i.e., a resolution of the singularity Spec (A). Denote by m' the maximal ideal of A. Let $\pi^{-1}(m') = Y_1 \cup \cdots \cup Y_d$, where the Y_i are distinct irreducible curves on Y. Then, according to Artin [1, page 132] there is a unique smallest positive divisor Z, with support $\bigcup_{i=1}^{d} Y_i$, such that $Z \cdot Y_i \leqq 0$ for all i. Z is called the fundamental divisor. We also have the divisor of the maximal ideal, M, given by

$$M = \sum_{i=1}^{d} m_i Y_i \ ,$$

where $m_i = \min_{t \in m'} \{w_i(t)\}$ and w_i is the valuation determined by $Y_i \subseteqq Y$. In general $Z \leqq M$. Artin [1, Theorem 4] shows that if Spec (A) has a rational singularity, then $Z = M$ on every resolution. Laufer [4, Theorem 3.13] proves that if Spec (A) has a minimally elliptic double point singularity, then $Z = M$ on every resolution. Laufer also gives examples of double point singularities for which $Z < M$. His surfaces have defining equation $z^2 = f(x, y)$, where $f(x, y) \in k[[x, y]]$, $f(0, 0) = 0$, and $f(x, y)$ is reducible at $(0, 0)$.

In this paper we show that if $f(x, y)$ has even order or if $f(x, y)$ has odd order and is irreducible at $(0, 0)$, then $Z = M$ on the minimal resolution of $z^2 = f(x, y)$. In §1 we give a method for obtaining a specific resolution of Spec (A) [3]. In §2 we perform some necessary computations with Z and M, and in §3 we give the proofs of the theorems.

1. Methods for resolving double point singularities. Let A