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RESTRICTIONS OF CERTAIN FUNCTION SPACES TO CLOSED SUBGROUPS OF LOCALLY COMPACT GROUPS

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Let G be a locally compact group, and $E(G)$ either the space $C_u(G)$ of bounded left and right uniformly continuous functions on G, the space $W(G)$ of weakly almost periodic functions on G, or the Fourier-Stieltjes algebra $B(G)$ of G. Let $E(G)|_H$ be the space of restrictions of $E(G)$-functions to the closed subgroup H of G. A necessary and sufficient condition is given for an $E(H)$-function to belong to $E(G)|_H$ when H is a normal subgroup of G. It is also shown that $E(G)|_H$ is all of $E(H)$ when H is any closed subgroup of a [SIN]-group. The techniques employed here can be used to deal with other function spaces.

Let $C(G)$ be the space of bounded continuous complex-valued functions on G with the uniform norm, $\| \quad \|_\infty$, and $\beta(G)$ be the Stone-Cech compactification of G, i.e., the maximal ideal space of $C(G)$, to which $C(G)$-functions extend naturally, via the Gelfand transform. The left translation operator is denoted λ:

$$[\lambda(g)u](g') = u(g^{-1}g') \qquad g, g'G, u \in C(G) .$$

The reader will recall that u in $C(G)$ is called weakly almost periodic if the set $\lambda(G)u$ of left translates of u is relatively compact in the weak topology of $C(G)$. Equivalently, one may require that for any sequence $\{g_j\}$ of elements of G there is a subsequence $\{g'_j\}$ such that $\lambda(g'_j)u$ converges weakly in $C(G)$, or such that $\lambda(g'_j)u$ converges pointwise on $\beta(G)$. This and other results on weakly almost periodic functions are summarized in [1]. The space $W(G)$ of weakly almost periodic functions in $C(G)$ is given the uniform norm.

The Fourier-Stieltjes algebra $B(G)$ of G is the algebra of coordinate functions

$$u: g \longrightarrow \langle \pi(g)\xi, \eta \rangle \qquad \xi, \eta \in \mathscr{H}_\pi$$

of continuous unitary representations π of G on Hilbert spaces $\mathscr{H}_\pi$; $B(G)$ is normed thus:

$$\|u\|_B = \min \{\|\xi\|_{\mathscr{H}_\pi} \cdot \|\eta\|_{\mathscr{H}_\pi}: u = \langle \pi\xi, \eta \rangle\} .$$

The basic facts about $B(G)$ can be found in [4]. We recall from [1] that