PACIFIC JOURNAL OF MATHEMATICS
Vol. 80, No. 1, 1979

SPECTRAL SYNTHESIS IN SEGAL ALGEBRAS ON HYPERGROUPS

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Warner (1966), Hewitt and Ross (1970), Yap (1970), and Yap (1971) extended the so-called Ditkin's condition for the group algebra $L^1(G)$ of a locally compact abelian group G to the algebras $L^1(G) \cap L^2(G)$, dense subalgebras of $L^1(G)$ which are essential Banach $L^1(G)$-modules, $L^1(G) \cap L^p(G)(1 \leqq p < \infty)$ and Segal algebras respectively. Chilana and Ross (1978) proved that the algebra $L^1(K)$ satisfies a stronger form of Ditkin's condition at points of the center $Z(\hat{K})$ of $\hat{K}$, where K is a commutative locally compact hypergroup such that its dual $\hat{K}$ is also a hypergroup under pointwise operations. Topological hypergroups have been defined and studied by Dunkl (1973), Spector (1973), and Jewett (1975) to begin with. In this paper we define Segal algebras on K and prove that they satisfy a stronger form of Ditkin's condition at the points of $Z(\hat{K})$. Examples include the analogues of some Segal algebras on groups and their intersections.

1. Introduction. In this paper we define and study Segal algebras on hypergroups with emphasis on spectral synthesis. A good deal of Harmonic Analysis has recently been developed on locally compact hypergroups by Dunkl [5], Spector [21], Jewett [9], and Ross ([17], [18]). Our basic reference will be Jewett [9]. Throughout this paper K will denote a commutative locally compact hypergroup ('Convos' in [9]) such that its dual $\hat{K}$ is a hypergroup under pointwise operations and notation and terminology for Harmonic Analysis on K will be as in [4]. As proved in ([23], Appendix) K is first countable if and only if it is metrizable. Being commutative, K admits a Haar measure m, as shown by Spector [22]. The convolution algebra $L^1(m) = L^1(K)$ can be identified with the pointwise algebra $A(\hat{K})$ of Fourier transforms on $\hat{K}$. Chilana and Ross [4] proved that $A(\hat{K})$ is a regular algebra of functions on $\hat{K}$ with a bounded approximate unit and it satisfies a stronger form of Ditkin's condition at points in the center $Z(\hat{K})$ of $\hat{K}$. They also gave examples to show that not all points in $\hat{K}$ need be spectral sets. This is partially in contrast with the situation in locally compact abelian groups where Ditkin's condition is satisfied for $L^1(G)$ at each point of $\hat{G}$. Warner [24] proved it for the algebra $L^1(G) \cap L^2(G)$, Hewitt and Ross ([7], 39.32) showed that it is true for dense Banach modules $\mathscr{U}$ in $L^1(G)$ such that $\mathscr{U} * L^1(G)$ is dense in $\mathscr{U}$. Also Yap [26] proved the same for the algebra $L^1(G) \cap L^p(G)(1 \leqq p < \infty)$ and then extended it to Segal